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A Kind of Non-associative Groupoids and Quasi Neutrosophic Extended Triplet Groupoids (QNET-Groupoids)

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Abstract: The various generalized associative laws can be considered as generalizations of traditional symmetry. Based on the theories of CA-groupoid, TA-groupoid and neutrosophic extended triplet (NET), this paper first proposes a new concept, which is type-2 cyclic associative groupoid (shortly by T2CA-groupoid), and gives some examples and basic properties. Furthermore, as a combination of neutrosophic extended triplet group (NETG) and T2CA-groupoid, the notion of type-2 cyclic associative neutrosophic extended triplet groupoid (T2CA-NET-groupoid) is introduced, and a decomposition theorem of T2CA-NET-groupoid is proved. Finally, as a generalization of neutrosophic extended triplet group (NETG), the concept of quasi neutrosophic extended triplet groupoid (QNET-groupoid) is introduced, and the relationships among T2CA-QNET-groupoid, T2CA-NET-groupoid and CA-NET-groupoid are discussed.

Keywords: Semigroup; Type-2 cyclic associative groupoid (T2CA-groupoid); neutrosophic extended triplet group (NETG); decomposition theorem; quasi neutrosophic extended triplet groupoid (QNET-groupoid)

1. Introduction

Groups and semigroups ([1–5, 7]) are essential branches of algebra, with the development of semigroup, the study of generalized semigroup has become an important topic. As far as we know the term groupoid (also called a magma) consists of a set G equipped with a binary operation. Despite the lack of further axioms, interesting results about groupoids exist [6].

The theory of non-associative algebras has seen new impetuous developments in recent years. Starting from algebraic topology, geometry and physics, new non-associative structures have emerged, such as triple systems, pairs, coalgebras and superalgebras. From a purely algebraic point of view, these structures are interesting. They have produced innovative ideas and methods that can help solve some algebraic problems. In fact, various generalized association identities are studied in many branches, for examples, functional equations [8-9], non-associative algebras [10], image processing [11] theory and networks [12].

The term "cyclic associative law" first appeared in the paper [13] published in 1954, which means an equation in the axiomatic system of Boolean algebra obtained in the literature [14] in 1946, namely $(ab)c=(bc)a$. Later, references [15-18] studied the relevant algebraic structures satisfying the cyclic binding law, however, the cyclic associative law in these references is actually a dual form of the cyclic associative law in [13-14], which is, $x(yz)=z(xy)$. In [19], we introduce the notion of formal cyclic associative groupoid (CA-groupoid), and systematically study its properties and the relationship between CA-groupoid and neutrosophic extended triplet group (NETG).

Moreover, in some literatures ([7-9, 20]), the cyclic associative law is also used to refer to the following equation:

$$x(yz)=(zx)y.$$

This meaning first appeared in Hosszü's study of function equation [20]. In this way, the term "cyclic associative law" has at least two different meanings in historical documents.

In order to avoid confusion, the equation $x(yz)=(zx)y$ is called type-2 cyclic associative law in this paper, and we focus on the basic properties and structures of the groupoid satisfying type-2 cyclic associative law, calling it type-2 cyclic associative groupoid.

In addition, Smarandache first proposed the new notion of neutrosophic extended triplet group (NETG) in [21], and many other significant results on NETGs and related algebraic systems can be found in [22-25]. In this paper, we analyze the structure of type-2 cyclic associative neutrosophic extended triplet groupoid (T2CA-NET-Groupoid) and study the relationship with the commutative regular semigroup.

This paper is organized as follows. In Section 2, we show some significant concepts and basic properties of groupoid, CA-groupoid and neutrosophic extended triplet groupoid (NETG). In Section 3, we put forward the concept of type-2 cyclic associative groupoid (T2CA-groupoid), and show some typical examples. In Section 4, we discuss the basic properties of the T2CA-groupoid and show some important results on cancellative T2CA-Groupoids. In Section 5, we introduce an important class of T2CA-groupoids for the first time, and we call it a type-2 cyclic associative neutrosophic extended triplet groupoid (T2CA-NET-groupoid). We first study its basic properties, and then gets its decomposition theorem, and finally, we study the relationship between T2CA-NET-groupoid and commutative regular semigroup. In Section 6, we introduce another significant class of groupoids. We call it a quasi neutrosophic extended triplet groupoid (QNET-groupoid) and further discuss the relationship between T2CA-QNET-groupoid, QNETG, T2CA-NET-groupoid, and CA-NET-groupoid. In Section 7, we present the summary and plans for future work.

2. Preliminaries

We give some notions and results about groupoids in this section.

A groupoid refers to an algebraic structure composed of non-empty sets, on which binary operations * are acted. Traditionally, when the * operator is omitted, it will not be confused. Assume $(S, *)$ is a groupoid, we show some concepts as follows:

(1) An element $x\in S$ is called idempotent if $x^2=x$.

(2) An element $x\in S$ is right cancellative (respectively left cancellative), if for all $y, z\in S$, $y*x = z*x \Rightarrow y = z$ ($x*y = x*z \Rightarrow y = z$). If an element both right and left cancellative, then it is cancellative. S is called right cancellative (left cancellative, cancellative), if each element of S is right cancellative (left cancellative, cancellative).

(3) If for any $x, y, z\in S$, $x*(y*z)=(x*y)*z$, S is called semigroup. A semigroup $(S, *)$ is commutative, if for all $x, y\in S$, $x*y=y*x$.

(4) If $\forall x\in S$, $x^2=x$, we call the semigroup $(S, *)$ as a band.

Definition 1. *([18, 26])* Let $(S, *)$ be a groupoid, for any $x, y, z\in S$.

(1) If $x*(y*z) = z*(x*y)$, then S is called a cyclic associative groupoid (or shortly CA-groupoid).
(2) If $(x*y)*z = (z*y)*x$, then S is called a CA-AG-groupoid.

Proposition 1. *[19]* If $(S, *)$ is a CA-groupoid ($\forall r, s, t, u, v, w\in S$), then:

(1) $(r*s)*(t*u) = (u*r)*(t*s)$;

(2) $(r*s)*((t*u)*(v*w)) = (u*r)*((t*s)*(v*w))$.

Definition 2. ([*21,27]*) Let *S* be a non-empty set, and * is a binary operation on *S*. S is called a neutrosophic extended triplet set, if for each $x \in$ S, there is a neutral "*x*" (denote by neut(*x*)), and the opposite of "*x*" (denote by *anti(x)*), such that *x*neut(x) = neut(x)*x = x, x*anti(x) = anti(x)*x = neut(x).*

The set of *neut(a)* and *anti(a)* is represented by the notations *{neut(a)}* and *{anti(a)}*; any certain one of *neut(a)* and *anti(a)* is represented by us with *neut(a)* and *anti(a)*.

Definition 3. *([21, 27])* Let *(S, *)* be a neutrosophic extended triplet set. If
*(1) (S, *)* is well-defined, *i.e., x*y∈S (∀ x, y∈S).*
*(2) (S, *)* is associative, *i.e., (x * y) * z = x * (y * z) (∀ x, y, z∈S).*
Then, *(S, *)* is called a neutrosophic extended triplet group (NETG). If *x*y=y*x (∀ x, y∈S), S i*s called a commutative NETG.

Proposition 2. *([23, 24])* If *(S, *)* is a NETG, then *(∀x∈S) neut(x)* is unique.

Theorem 1. *([19])* Let *(S, *)* be a TA-NET-groupoid. Denote the set of all different neutral element in *S* by *N(S)*. Put *S(e) = {x∈S | neut(x) = e} (∀e∈N(S)),* then *S*(*e*) is a subgroup of *S*.

Theorem 2. ([28]) Assume that (*S*, *) is a CA-groupoid, the following statements are equivalent:
(1) *S* is a CA-NET-groupoid;
(2) *S* is a CA- (r, l)-NET-groupoid;
(3) *S* is a CA- (r, r)-NET-groupoid;
(4) *S* is a CA- (l, r)-NET-groupoid;
(5) *S* is a CA- (l, l)-NET-groupoid;
(6) *S* is a commutative regular semigroup.

3. Type-2 Cyclic Associative Groupoids (T2CA-Groupoids)

Definition 4. Let *(S, *)* be a groupoid, for any *r, s, t∈S.* If

$$r*(s*t) = (t*r)*s,$$

then *(S, *)* is called a type-2 cyclic associative groupoid (shortly, T2CA-groupoid).

The following example shows that there is T2CA-groupoid, which is not a CA-groupoid, not a semigroup, not an AG-groupoid. Obviously, it is not a CA-AG-groupoid.

Example 1. Put *S* = {1, 2, 3, 4, 5, 6, 7 ,8}, and define the operations * on *S* as shown in Table 1. Then (*S*, *) is a T2CA-groupoid. We can verify that (*S*, *) is not a semigroup, due to the fact that (6*7)*7 = 2 ≠ 1 = 6*(7*7); (*S*, *) is not a CA-groupoid, because 6*(6*7) = 1 ≠ 2 = 7*(6*6); (S, *) is not an AG-groupoid, since (6*7)*7 = 2 ≠ 1 = (7*7)* 6. Obviously**,** (*S*, *) is not a CA-AG-groupoid.

Table 1. The operation * on *S*

*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
3	1	1	1	2	1	1	2	1
4	1	1	1	1	1	1	1	1
5	1	1	1	1	1	2	2	1
6	1	1	5	1	1	4	5	1
7	1	1	2	2	1	4	2	2
8	1	1	2	1	1	1	2	2

From the following example, we know that there is T2CA-groupoid which is a semigroup but not commutative.

Example 2. Table 2 shows the non-commutative T2CA-groupoid of order 6, and (*S*, *) is a semigroup.

Table 2. Cayley table on $S = \{r, s, t, u, v, w\}$

*	**r**	**S**	**t**	**u**	**v**	**w**
r	r	R	t	t	t	t
s	r	R	t	t	t	t
t	t	T	r	r	r	r
u	t	t	r	r	r	s
v	t	t	r	s	s	r
w	t	t	r	r	s	r

Proposition 3. (1) Each commutative semigroup is a T2CA-groupoid. (2) Let (*S*, *) be a T2CA-groupoid. If *S* is commutative, then *S* is a commutative semigroup.

Proof. By Definition 4, this is obvious. □

Definition 5. Let (*S*, *) is a T2CA-groupoid, then an element *e* in S is called the quasi left identity element if for all *a* in *S*, $e*a = a\ (a \neq e)$; and it is called the quasi right identity element for all *a* in *S*, $a*e = a\ (a \neq e)$. If *e* is both quasi left and right identity element**,** it is called quasi identity element**.**

Example 3. As shown in Table 3, put $S = \{f, g, h, j, k\}$, and define the operations * on *S*. Then we can verify through MATLAB that (*S*, *) is a T2CA-groupoid, and *f* is the quasi identity element in *S*, due to the fact that $f*g = g*f = g, f*h = h*f = h, f*j = j*f = j, f*k = k*f = k.$

Table 3. The operation * on *S*

*	***f***	***g***	***h***	***j***	***k***
f	G	g	h	j	k
g	G	g	h	j	k
h	H	h	g	j	k
j	J	j	j	j	j
k	k	k	k	j	k

Theorem 3. Let (*S*, *) be a T2CA-groupoid with quasi identity element *e*, that is $\forall x \in S, e*x = x*e = x\ (x \neq e)$. Then *S* is commutative.

Proof. For any $x, y \in S$, when $x = y$, obviously $x*y = y*x$. Suppose $x \neq y$, we have:

(1) Assume that $y = e$, due to $x \neq y$, then $x \neq e$. Therefore, $e*x = x = x*e$. That is, $x*y = y*x$.

(2) Suppose $x \neq e$, and $y \neq e$, there are:

Case 1, if $x*y \neq e$, by Definition 4, we can get that $x*y = e*(x*y) = (y*e)*x = y*x$.

Case 2, if $x*y = e$, we have $y*x = e$. Otherwise, suppose $y*x \neq e$, by Definition 4 we have $y*x = e*(y*x) = (x*e)*y = x*y = e$. This contradicts $y*x \neq e$.

Hence**,** *S* is commutative. □

Theorem 4. Let (*S*, *) be a T2CA-groupoid, $e \in S$.

(1) If *e* is the quasi left identity element of *S*, that is, $\forall x \in S, e*x = x\ (x \neq e)$, then *e* is the quasi right identity element.

(2) If *e* is the quasi right identity element of *S*, that is, $\forall x \in S, x*e = x\ (x \neq e)$, then *e* is the quasi left identity element.

Proof. (1) If *e* is the quasi left identity element of *S*. For each $x \in S, e*x = x\ (x \neq e)$, we have $x*e = (e*x)*e = x*(e*e)$, and

$$x = e*x = e*(e*x) = (x*e)*e = (x*(e*e))*e = (e*e)*(e*x) = (e*e)*x = e*(x*e).$$

Case 1, when $x*e \neq e$, from $x = e*(x*e)$ and definition of quasi left identity element, we can get that $x = x*e$.

Case 2, when $x*e = e$, $x = e*(x*e) = e*e$, then $e*e \neq e$. Otherwise, if $e*e = e$, we have $x = e*e = e$. This contradicts $x \neq e$. Due to that e is the quasi left identity element of S, therefore, $e*(e*e) = e*e$. And,

$$x*e = x*(e*e) = x*(e*(e*e)) = (e*e)*(x*e) = (e*e)*e = e*(e*e) = e*e = x.$$

Hence, e is the quasi right identity element.

(2) Suppose that e is the quasi right identity element of S. For any $x\in S$, $x*e = x$ $(x \neq e)$, we have $ex = e*(x*e) = (e*e)*x$, and

$$x = x*e = (x*e)*e = e*(e*x) = e*((e*e)*x) = (x*e)*(e*e) = x*(e*e) = (e*x)*e.$$

Case 1, when $e*x \neq e$, from definition of quasi right identity element, we can get that $(e*x)*x = e*x$, so $x = (e*x)*e = e*x$.

Case 2, when $e*x = e$, $x = (e*x)*e = e*e$, then $e*e \neq e$. Otherwise, if $e*e = e$, we have $x = e*e = e$. This contradicts $x \neq e$. Due to that e is the quasi right identity element of S, therefore, $e*(e*e) = e*e$. Moreover,

$$\begin{aligned} e*x &= (e*e)*x = x*x \quad (\text{Applying } x = e*e) \\ &= x*(e*e) = x. \end{aligned}$$

Hence, e is the quasi left identity element. □

4. Some Properties of Type-2 Cyclic Associative Groupoids (T2CA-Groupoids)

Proposition 4. Let $(S, *)$ be a T2CA-groupoid. Then,

(1) $\forall a, b, c, d\in S$, $(a*b)*(c*d) = (b*a)*(d*c)$;

(2) $\forall a, b, c, d, e, f\in S$, $(a*b)*[(c*d)*(e*f)] = [(b*f)*(c*a)]*(e*d)$.

Proof. (1) Suppose $(S, *)$ is a T2CA-groupoid, then for any $a, b, c, d, e, f\in S$, by Definition 4 we have

$$(a*b)*(c*d) = [d*(a*b)]*c = [(b*d)*a]*c = a*[c*(b*d)] = a*[(d*c)*b] = (b*a)*(d*c).$$

(2) For any $a, b, c, d, e, f\in S$, by Definition 4 we have

$$\begin{aligned} (a*b)*[(c*d)*(e*f)] &= (a*b)*[(d*c)*(f*e)] \qquad (\text{By } (a*b)*(c*d) = (b*a)*(d*c)) \\ &= b*[((d*c)*(f*e))*a] = b*[(f*e)*(a*(d*c))] = b*[(f*e)*((c*a)*d)] \\ &= b*[(d*(f*e))*(c*a)] = [(c*a)*b]*(d*(f*e)) = [a*(b*c)]*((e*d)*f) \\ &= [f*(a*(b*c))]*(e*d) = [f*((c*a)*b)]*(e*d) = [(b*f)*(c*a)]*(e*d). \ \square \end{aligned}$$

Theorem 5. Suppose $(S, *)$ is a T2CA-groupoid.

(1) If $\forall k\in S$, $\exists\ e \in S$ such that $e*k = k$, that is, S have a left identity element, then S is a commutative semigroup.

(2) If $\forall k\in S$, $\exists e \in S$ such that $k*e = k$, that is, S have a right identity element, then S is a commutative semigroup.

(3) If $e \in S$ is a left identity element, then e is an identity element.

(4) If $e \in S$ is a right identity element, then e is an identity element.

Proof. (1) Suppose $(S, *)$ is a T2CA-groupoid. $\forall k, w \in S$, we have

$$k*w = [e*(e*k)]*w = [(k*e)*e]*w = e*[w*(k*e)] = e*[(e*w)*k] = e*(w*k) = w*k.$$

Therefore, $(S, *)$ is a commutative T2CA-groupoid. Applying Proposition 3 (2), we get that $(S, *)$ is a commutative semigroup.

(2) Suppose $(S, *)$ is a T2CA-groupoid. $\forall\ k, w \in S$, there are:

$$\begin{aligned} k*w &= [e*(e*k)]*w = k*[(w*e)*e] = (e*k)*(w*e) \\ &= (k*e)*(e*w) \qquad (\text{By Proposition 4 (1)}) \\ &= [w*(k*e)]*e = (w*k)*e = w*k. \end{aligned}$$

Therefore, $(S, *)$ is a commutative T2CA-groupoid. Applying Proposition 3 (2), we get that $(S, *)$ is a commutative semigroup.

(3) If e is a left identity element in S. $\forall\ k \in S$, applying Proposition 4 (1) there are:

$$k = e*k = e*(e*k) = (k*e)*e = (k*e)*(e*e) = (e*k)*(e*e) = k*e.$$

Thus, $e \in S$ is an identity element.

(4) If e is a right identity element in S, $\forall\ k \in S$, applying Proposition 4 (1) we get

$$k = k*e = (k*e)*e = e*(e*k) = (e*e)*(e*k) = (e*e)*(k*e) = e*k.$$

Therefore, $e \in S$ is an identity element. □

Definition 6. Suppose S is a T2CA-groupoid. S is called a left cancellative (right cancellative, cancellative) T2CA-groupoid, if each element of S is left cancellative (right cancellative, cancellative).

Theorem 6. Suppose (S, *) is a T2CA-groupoid, $\forall p, q\in S$:

(1) if p is right cancellative or left cancellative, then p is cancellative;

(2) if p is right cancellative and q is left cancellative, then $p*q$ is cancellative;

(3) if $p*q$ is right cancellative, then $p*q = q*p$;

(4) if $p*q$ is cancellative, then p and q are cancellative;

(5) if p and $p*q$ are right cancellative, then $p*q$ is cancellative.

Proof. Let (S, *) be a T2CA-groupoid, $p, q\in S$.

(1) If p is a right cancellative element, $p*k = p*w$ ($\forall\ k, w\in S$), using type-2 cyclic association:

$$(k*p)*p = p*(p*k) = p*(p*w) = (w*p)*p.$$

Applying right cancellation property of p two times, then $k = w$. Therefore, $p\in S$ is a left cancellative element, so p is a cancellative element in S.

Similarly, if p is a left cancellative element, $k*p = w*p$ ($\forall k, w\in S$), using type-2 cyclic association:

$$p*(p*k) = (k*p)*p = (w*p)*p = p*(p*w).$$

Using left cancellation property of p two times, then $k = w$. Therefore, $p\in S$ is a right cancellative element, so p is a cancellative element in S.

(2) If p is right cancellative, q is left cancellative, $k*(p*q) = w*(p*q)$ ($\forall k, w\in S$), using type-2 cyclic association:

$$(q*k)*p = k*(p*q) = k*(p*q) = w*(p*q) = w*(p*q) = (q*w)*p.$$

Since p is right cancellative, q is left cancellative, we get $k = w$. Therefore, $p*q$ is a right cancellative.

Moreover, if $(p*q)*k = (p*q)*w$ ($\forall k, w\in S$), we have:

$$q*(k*p) = (p*q)*k = (p*q)*k = (p*q)*w = q*(w*p).$$

Since p is right cancellative, q is left cancellative, we get $k = w$. Therefore, $p*q$ is a left cancellative. Hence, $p*q$ is cancellative.

(3) Suppose $p*q$ is right cancellative. By Proposition 4 (2), we have:

$$[(p*q)*(q*p)] * (p*q) = (q*p)*[(p*q)*(p*q)] = [(p*q)*(p*q)]*(p*q).$$

Since $p*q$ is right cancellative, then $(p*q)*(q*p) = (p*q)*(p*q)$. Applying Proposition 4 (1), we get that $(q*p)*(p*q) = (p*q)*(p*q)$.Moreover, since $p*q$ is right cancellative, then $q*p = p*q$.

(4) Suppose $p*q$ is cancellative. If $q*k = q*w$ ($\forall k, w\in S$), there are:

$$k*(p*q) = (q*k)*p = (q*w)*p = w*(p*q).$$

Since $p*q$ is cancellative, so $k=w$. This means that q is left cancellative. According to (1), we know q is cancellative.

And, since $p*q$ is cancellative, then $p*q$ is right cancellative, according to (5) we get $q*p = p*q$. *So*, $q*p$ is cancellative, p is cancellative. Therefore, if $p*q$ is cancellative, then p and q are cancellative.

(5) Assume that p and $p*q$ are right cancellative. If $(p*q) * k = (p*q) * w$ ($\forall k, w \in S$), using type-2 cyclic association:

$$(k*p)*(p*q) = p*((p*q)*k) = p*((p*q)*w) = (w*p)*(p*q).$$

Since $p*q$ is right cancellative, so $k*p = w*p$. Moreover, p is right cancellative, so $k = w$. Thus, $p*q$ is left cancellative, according to (1), we know $p*q$ is cancellative. □

According to Theorem 6, we have the following corollary.

Corollary 1. Suppose $(S, *)$ is a T2CA-groupoid, then the following asserts are equivalent:

(i) S is a left cancellative T2CA-groupoid;

(ii) S is a right cancellative T2CA-groupoid;

(iii) S is a cancellative and commutative semigroup;

Proof. (i) ⇒ (ii): Follow Theorem 6 (1).

(ii) ⇒ (iii): Assume that S is right cancellative, by using Theorem 6 (1), we get S is cancellative. For any $p, q \in S$, according to Theorem 6 (3), we have $p*q = q*p$, then S is commutative. When applying Proposition 3 (2), we get that S is a commutative semigroup. Therefore, S is a cancellative and commutative semigroup.

(iii) ⇒ (i): Obviously. □

Corollary 2. Let $(S, *)$ be a T2CA-groupoid. If there exists a cancellative element in S, then the set $M = \{p \in S: p \text{ is cancellative}\}$ is a sub T2CA-groupoid of S.

Proof. Through the existence of a condition for cancellative elements in S, we get that M is not empty. $\forall p, q \in M$, p and q are right and left cancellative. By Theorem 6 (2), we get $p*q$ is cancellative. Thus $p*q \in M$. Therefore, M is a sub T2CA-groupoid of S. □

Corollary 3. Let $(S, *)$ be a T2CA-groupoid. If there exists a non-cancellative element in S, then the set $N = \{p \in S: p \text{ is non-cancellative}\}$ is a sub T2CA-groupoid of S.

Proof. Obviously, N is non-empty. $\forall\ p, q \in N$, p and q are non-cancellative. Through Theorem 6 (4), we know that $p*q$ is non-cancellative. Thus, $p*q \in N$. Therefore, N is a sub T2CA-groupoid of S.□

Theorem 7. Suppose $(S, *)$ is a T2CA-groupoid, $r, s, t \in S$. Define on S the relation ~ as:

$$r \sim s \Leftrightarrow r \text{ and } s \text{ are both cancellative or non-cancellative.}$$

Then ~ is an equivalence relation.

Proof. Obviously, ~ is reflexive and symmetric.

Next, Assume $r \sim s$ and $s \sim t$. If r and s are non-cancellative, from $s \sim t$ we get t is non-cancellative, thus r and t are non-cancellative, i.e., $r \sim t$; if r and s are cancellative, from $s \sim t$ we get t is cancellative, thus r and t are cancellative, i.e., $r \sim t$. Thus ~ is transitive.

Therefore, ~ is an equivalence relation. □

Definition 7. Let $(S_1, *_1)$, $(S_2, *_2)$ be two T2CA-groupoids, $S_1 \times S_2 = \{(p, q) \mid p \in S_1, q \in S_2\}$. Define binary operation * on $S_1 \times S_2$ as following:

$$(p_1, p_2) * (q_1, q_2) = (p_1 *_1 q_1, p_2 *_2 q_2), \text{ for any } (p_1, p_2), (q_1, q_2) \in S_1 \times S_2.$$

S_1 and S_2 are called the direct factors of $S_1 \times S_2$, and $(S_1 \times S_2, *)$ is called the direct product of $(S_1, *_1)$ and $(S_2, *_2)$.

Theorem 8. Let $(S_1, *_1)$, $(S_2, *_2)$ be two T2CA-groupoids. Then the direct product $(S_1 \times S_2, *)$ is a T2CA-groupoid.

Proof. Let (r_1, r_2), (s_1, s_2), $(t_1, t_2) \in S_1 \times S_2$. We have:

$$\begin{aligned}(r_1, r_2) * ((s_1, s_2) * (t_1, t_2)) &= (r_1, r_2) * (s_1 *_1 t_1, s_2 *_2 t_2)\\ &= (r_1 *_1 (s_1 *_1 t_1), r_2 *_2 (s_2 *_2 t_2)) = ((t_1 *_1 r_1) *_1 s_1), (t_2 *_2 r_2) *_2 s_2))\\ &= (t_1 *_1 r_1, t_2 *_2 r_2) * (s_1, s_2) = ((t_1, t_2) * (r_1, r_2)) * (s_1, s_2).\end{aligned}$$

Hence, $(S_1 \times S_2, *)$ is a T2CA-groupoid. □

Theorem 9. Let S_1, S_2 are T2CA-groupoids, if a and b are cancellative, then $(p, q) \in S_1 \times S_2$ is cancellative $(p \in S_1, q \in S_2)$.

Proof. Applying Theorem 8, we know $S_1 \times S_2$ is a T2CA-groupoid. Assume p and q are cancellative $(p \in S_1, q \in S_2)$, for any (x_1, x_2), $(y_1, y_2) \in S_1 \times S_2$, $(p, q)*(x_1, x_2) = (p, q)*(y_1, y_2)$. Then

$$(px_1, qx_2) = (py_1, qy_2); px_1 = py_1, qx_2 = qy_2.$$

And, according to p and q are cancellative, we get that $x_1 = y_1$, $x_2 = y_2$. That is $(x_1, x_2) = (y_1, y_2)$. Hence, (p, q) is cancellative. □

5. Type-2 Cyclic Associative Neutrosophic Extended Triplet Groupoids (T2CA-NET-Groupoids)

In this section, we first proposed an important class of T2CA-groupoids, namely T2CA-NET-groupoids. After giving the basic definitions and properties, this section focuses on the structure of T2CA-NET-groupoids, and the relationship between T2CA-NET-groupoids and commutative regular semigroups. Fortunately, we got very exciting results.

Definition 8. Let $(S, *)$ be a neutrosophic extended triplet set. $(S, *)$ is called a type-2 cyclic associative neutrosophic extended triplet groupoid (shortly, T2CA-NET-groupoid), if the following conditions are satisfied:
(1) $(S, *)$ is well-defined, i.e., $\forall a, b \in S$, one has $a*b \in S$.
(2) $(S, *)$ is type-2 cyclic associative, i.e., $a*(b*c) = (c*a)*b$, $\forall a, b, c \in S$.
S is called a commutative T2CA-NET-groupoid if $a*b = b*a$, $\forall a, b \in S$.

Theorem 10. Let $(S, *)$ be a T2CA-NET-groupoid, $\forall x \in S$, then $neut(x)$ is unique.

Proof. We assume that local unit element $neut(x)$ is not unique in S. Then, there is $s, t \in \{neut(x)\}$ such that $(p, q \in S)$

$$x*s = s*x = x \text{ and } x*p = p*x = s; x*t = t*x = x \text{ and } x*q = q*x = t.$$

(1) To prove $s = s*t$. Due to the fact

$$s = p*x = p*(t*x) = (x*p)*t = s*t.$$

(2) To prove $t = t*s$. Due to the fact

$$t = q*x = q*(s*x) = (x*q)*s = t*s.$$

(3) To prove $s = s*s$. Due to the fact

$$s = p*x = p*(s*x) = (x*p)*s = s*s.$$

(4) To prove $t*s = s*t$. Due to the fact

$$t*s = (t*s)*s = s*(s*t) = s*s = s = s*t.$$

Hence $s = t$, and $neut(x)$ is unique in S. □

Remark 1. In a T2CA-NET-groupoid (S, *), we know from Example 4 that *anti(x)* may be not unique.

Example 4. Let $S = \{g, k, u, v, w\}$. The operate * on S is defined as Table 4. Then, $(S, *)$ is T2CA-NET-groupoid. Moreover, $neut(g) = g$, and $\{anti(g)\} = \{g, k, u, v, w\}$.

Table 4. The operation * on S

*	g	k	u	v	w
g	g	g	g	g	g
k	g	k	u	v	w
u	g	u	w	k	v
v	g	v	k	w	u
w	g	w	v	u	k

Proposition 5. Suppose (S, *) is a T2CA-NET-groupoid. Then, for any $t \in S$,
(1) $neut(t)*neut(t) = neut(t)$;
(2) $neut(neut(t)) = neut(t)$;
(3) $anti(neut(t))*t = t$.

Proof. (1) Using $anti(t)*t = t*anti(t) = neut(t)$, we get

$$neut(t)*neut(t) = neut(t)*[anti(t)*t] = [t*neut(t)]*anti(t) = t*anti(t) = neut(t).$$

(2) According to the definition of $neut(neut(t))$ we can get:

$$neut(t)*neut(neut(t)) = neut(neut(t))*neut(t) = neut(t).$$

By the definition of $anti(neut(t))$ we can get:

$$neut(t)*anti(neut(t)) = anti(neut(t))*neut(t) = neut(neut(t)).$$

Applying (1) and Theorem 10, we get $neut(neut(t)) = neut(t)$.

(3) By Definition 4, Definition 8 and Proposition 5 (2), there are:

$$anti(neut(t))*t = anti(neut(t))*[t*neut(t)] = [neut(t)*anti(neut(t))]*t$$
$$= neut(neut(t))*t = neut(t)*t = t.$$

Therefore, $anti(neut(t))*t = t$. □

Remark 2. In a T2CA-NET-groupoid (S, *), we know from Example 5 that *neut(anti(t))* may be not equal to *neut(t)*.

Example 5. *Let $S = \{g, u, v, w\}$. The operate * on S is defined as Table 5. Then, (S, *) is T2CA-NET-groupoid. And,*

$$neut(g) = g,\ neut(u) = u,\ \{anti(g)\} = \{g, u, v, w\}.$$

While $anti(g) = u$, $neut(anti(g)) \neq neut(g)$, because $neut(anti(g)) = neut(u) = u \neq g = neut(g)$.

Table 5. The operation * on S

*	g	u	v	w
G	g	g	g	g
U	g	u	g	w
V	g	g	v	g
W	g	w	g	u

Theorem 11. Suppose (S, *) is a T2CA-NET-groupoid, then its idempotents are commutative.

Proof. If k, w an idempotent in S, then

$$(k*w)*(k*w) = (w*k)*(w*k) \qquad \text{(Using Proposition 4 (1))}$$
$$= [(w*k)*w]*k = [k*(w*w)]*k = (w*w)*(k*k) = w*k.$$

Moreover,

$$(k*w)*(k*w) = [w*(k*w)]*k = [(w*w)*k]*k = (w*k)*k$$
$$= k*(k*w) = (neut(k)*k)*(k*w)$$
$$= (k*neut(k))*(w*k) \quad \text{(Using Proposition 4 (1))}$$
$$= k*(w*k) = (k*k)*w = k*w.$$

Hence, we get $k*w = w*k$. That is, in a T2CA-NET-groupoid, its idempotents are commutative. □

Corollary 4. Every T2CA-NET-groupoid is commutative.

Proof. Suppose (S, *) is a T2CA-NET-groupoid. Applying Theorem 11, $neut(x)$ $(\forall x \in S)$ is idempotent. $\forall$ $k, w \in S$, we have

$$neut(k) * neut(w) = neut(w) * neut(k).$$

Further, for any $k, w \in S$, we have:

$$k*w = [k*neut(k)]*[w*neut(w)] = [neut(w)*(k*neut(k))]*w$$
$$= [(neut(k)*neut(w))*k]*w = k*[w*(neut(k)*neut(w))]$$
$$= k*[(neut(w)*w)*neut(k)] = k*[w*neut(k)]$$
$$= [neut(k)*k]*[w*neut(k)] = k*[(w*neut(k))*neut(k)]$$
$$= k*[neut(k)*(neut(k)*w)] = [(neut(k)*w)*k]*neut(k)$$
$$= [w*(k*neut(k))]*neut(k) = (w*k)*neut(k)$$
$$= (w*k)*[neut(k)*neut(k)]$$
$$= (k*w)*[neut(k)*neut(k)] \quad \text{(Applying Proposition 4 (1))}$$
$$= (k*w)*neut(k) \quad \text{(Applying Proposition 5 (1))}$$
$$= w*[neut(k)*k] = w*k.$$

Hence, every T2CA-NET-groupoid is commutative. □

Example 6. T2CA-NET-groupoid of order 5, given in Table 6, and

$$neut(a) = a, \{anti(a)\} = \{a, e\}; neut(b) = b, \{anti(b)\} = \{a, b, c, d, e\};$$
$$neut(c) = c, \{anti(c)\} = \{c, e\}; neut(d) = d, \{anti(d)\} = \{a, c, d, e\}; neut(e) = e, anti(e) = e.$$

Obviously, (S, *) is a commutative.

Table 6. Cayley table on $S = \{a, b, c, d, e\}$.

*	**a**	**b**	**c**	**d**	**e**
a	a	b	d	d	a
b	b	b	b	b	b
c	d	b	c	d	c
d	d	b	d	d	d
e	a	b	c	d	e

Proposition 6. Let (S, *) be a T2CA-NET-groupoid. Then for any $k \in S$, for all $t, u \in \{anti(k)\}$,

(1) $t*neut(k) = u*neut(k)$;
(2) $neut(u)*neut(k) = neut(k)*neut(u) = neut(k)$.
(3) $u*neut(k) \in \{anti(k)\}$;
(4) $u*neut(k) = (neut(k)*u)*neut(k)$;
(5) $u*neut(k) = neut(k)*u$;
(6) $neut(u*neut(k)) = neut(k)$.

Proof. (1) $\forall t, u \in \{anti(k)\}$, by the definition of opposite and neutral element, using Theorem 10, we get

$$k*t = t*k = neut(k), k*u = u*k = neut(k).$$
$$t*neut(k) = t*(u*k) = (k*t)*u = neut(k)*u = (k*u)*u = u*(u*k) = u*neut(k).$$

(2) $\forall u \in \{anti(k)\}$, by $k*u = u*k = neut(k)$, we have

$$neut(u)*neut(k) = neut(u)*(k*u) = [u*neut(u)]*k = u*k = neut(k);$$
$$neut(k)*neut(u) = (u*k)*neut(u) = k*[neut(u)*u] = k*u = neut(k).$$

That is, $neut(u)*neut(k) = neut(k)*neut(u) = neut(k)$ is true for all $k \in S$.

(3) $\forall k \in S$ and $u \in \{anti(k)\}$, $u*k=k*u=neut(k)$. Then, by Definition 4 and Proposition 5 (1), we have

$$k*[u*neut(k)] = [neut(k)*k]*u = k*u = neut(k);$$
$$[u*neut(k)]*k = neut(k)*(k*u) = neut(k)*neut(k) = neut(k).$$

This means that $u*neut(k) \in \{anti(k)\}$.

(4) $\forall k \in S$ and $u \in \{anti(k)\}$, $u*k = k*u = neut(k)$. Applying (1) and (3), we get

$$u*neut(k) = (u*neut(k))*neut(k).$$

On the other hand, by using Proposition 4 (1) and Proposition 5 (1), we get

$$[u*neut(k)]*neut(k) = [u*neut(k)]*[neut(k)*neut(k)]$$
$$= [neut(k)*u]*[neut(k)*neut(k)]= [neut(k)*u]*neut(k).$$

Combining two equations above, we get $u*neut(k) = [neut(k)*u]*neut(k)$.

(5) Assume that $u \in \{anti(k)\}$, then $k*u = u*k = neut(k)$, and $u*neut(u) = neut(u)*u = u$. By Proposition 5 (1) and (2), applying (2) and (3), there are

$$\begin{aligned}
neut(k)*u &= [neut(k)*neut(k)]*[u*neut(u)] = [neut(u)*(neut(k)*neut(k))]*u \\
&= [(neut(k)*neut(u))*neut(k)]*u = neut(k)*[u*(neut(k)*neut(u))] \\
&= neut(k)*[u*(neut(u)*neut(k))] = neut(k)*[(neut(k)*u)*neut(u)] \\
&= [neut(u)*neut(k)]*[neut(k)*u] = neut(k)*[neut(k)*u] \\
&= [u*neut(k)]*neut(k) \\
&= [neut(k)*u]*neut(k) \qquad \text{(By (3), } [u*neut(k)]*neut(k) = [neut(k)*u]*neut(k)\text{)} \\
&= u*neut(k). \qquad \text{(By (3), } u*neut(k) = [neut(k)*u]*neut(k)\text{)}
\end{aligned}$$

(6) Assume $u \in \{anti(k)\}$, denote $d = u*neut(k)$. We prove the following equations:

$$d*neut(k) = neut(k)*d = d;\ d*k = k*d = neut(k).$$

By Proposition 4 (1), Proposition 5 (1), and above (5), we get

$$\begin{aligned}
d*neut(k) &= [u*neut(k)]*neut(k) = [u*neut(k)]*[neut(k)*neut(k)] \\
&= [neut(k)*u]*[neut(k)*neut(k)] = [neut(k)*u]*neut(k) \\
&= u*[neut(k)*neut(k)] = u*neut(k) = d.
\end{aligned}$$

Using Definition 4 and (5), we have

$$neut(k)*d = neut(k)*[u*neut(k)] = neut(k)*[neut(k)*u] = [u*neut(k)]*neut(k) = d*neut(k) = d.$$

Moreover, using Proposition 5 (1), Definition 4, there are:

$$d*k = [u*neut(k)]*k = neut(k)*(k*u) = neut(k)*neut(k) = neut(k).$$
$$k*d = k*[u*neut(k)] = [neut(k)*k]*u = k*u = neut(k).$$

Thus,

$$d*neut(k) = neut(k)*d = d;\ d*k = k*d = neut(k).$$

According to the definition of neutral element and Theorem 10, we get $neut(k)$ is the neutral element of $d = u*neut(k)$. Hence, $neut(u*neut(k)) = neut(k)$. □

Theorem 12. Let $(S, *)$ be a T2CA-NET-groupoid. Put the set of all different neutral elements in S by $N(S)$, and $S(n) = \{a \in S \mid neut(a) = n\}$ $(\forall n \in N(S))$. Then:

(1) $S(n)$ is a subgroup of S;

(2) for any $n_1, n_2 \in N(S)$, $n_1 \neq n_2 \Rightarrow S(n_1) \cap S(n_2) = \varnothing$;

(3) $S = \bigcup_{n \in N(S)} S(n)$.

Proof. (1) For every $k \in S(n)$, $neut(k) = n$, we get that n is an identity element in $S(n)$. Applying Proposition 5 (1), there are $n*n = n$.

Assume $k, w \in S(n)$, then $neut(k) = neut(w) = n$. Next, we are going to prove that $neut(k*w) = n$.

Applying Definition 4, and Corollary 4, we have

$$(k*w)*n = y*(n*k) = w*k = k*w;$$
$$n*(k*w) = (w*n)*k = w*k = k*w.$$

Moreover, for any $anti(k)\in\{anti(k)\}$, $anti(w)\in\{anti(w)\}$. By using Definition 4 and Definition 8, we have

$$\begin{aligned}(k*w)*[anti(k)*anti(w)] &= w*[(anti(k)*anti(w))*k] = w*[anti(w)*(k*anti(k))]\\ &= w*(anti(w)*neut(k)) = [neut(k)*w]*anti(w) = (n*w)*anti(w)\\ &= w*[anti(w)*n] = w*anti(w) = neut(w) = n.\end{aligned}$$

$$\begin{aligned}[anti(k)*anti(w)]*(k*w) &= anti(w)*[(k*w)*anti(k)] = anti(w)*[w*(anti(k)*k)]\\ &= anti(w)*[w*neut(k)] = anti(w)*(w*n) = anti(w)*w\\ &= neut(w) = n.\end{aligned}$$

Thus, according to Theorem 10 and the definition of neutral elements, we get that $neut(k*w) = n$. Therefore, $k*w\in S(n)$, that is, $S(n)$ is closed under operation *.

Furthermore, $\forall k\in S(n)$, $\exists u\in S$ such that $u\in\{anti(k)\}$. Using Proposition 6(3), $u*neut(k)\in\{anti(k)\}$; and applying Proposition 6 (6), $neut(u*neut(k)) = neut(k)$.

Put $d = u*neut(k)$, we have

$$d = u*neut(k)\in\{anti(k)\},\ neut(d) = neut(u*neut(k)) = neut(k) = n.$$

Thus $d\in\{anti(k)\}$, $neut(d) = n$, *i.e.*, $d\in S(n)$ and d is the inverse element of k in $S(n)$.

Hence, $(S(n), *)$ is a subgroup of S.

(2) Suppose $k\in S(n_1)\cap S(n_2)$ and $n_1, n_2\in N(S)$. There are $neut(k)=n_1$, $neut(k)=n_2$. Applying Theorem 10, we get $n_1 = n_2$. Hence, $n_1\neq n_2 \Rightarrow S(n_1)\cap S(n_2) = \varnothing$.

(3) $\forall k\in S$, $\exists neut(k)\in S$. Put $n=neut(k)$, then $k\in S(n)$, $n\in N(S)$. This means that $S = \bigcup_{n\in N(S)} S(n)$. □

Example 7. T2CA-NET-groupoid of order 5, given in Table 7, and

$$neut(a) = a,\ anti(a) = a;\ neut(s) = a,\ anti(s) = s;$$
$$neut(d) = d,\ anti(d) = \{a, d, g\};\ neut(f) = d,\ anti(f) = \{s, f\};\ neut(g) = g,\ anti(g) = g.$$

Denote $S_1 = \{a, s\}$, $S_2 = \{d, f\}$, $S_3 = \{g\}$, then S_1, S_2 and S_3 are subgroup of S, and $S=S_1\cup S_2\cup S_3$, $S_1\cap S_2=\varnothing$, $S_1\cap S_3=\varnothing$, $S_2\cap S_3=\varnothing$.

Table 7. Cayley table on $S = \{a, s, d, f, g\}$.

*	***a***	***s***	***d***	***f***	***g***
a	*a*	*s*	*d*	*f*	*d*
s	*s*	*a*	*f*	*d*	*f*
d	*d*	*f*	*d*	*f*	*d*
f	*f*	*d*	*f*	*d*	*f*
g	*d*	*f*	*d*	*f*	*g*

Theorem 13. Suppose $(S, *)$ is a groupoid, then S is a T2CA-NET-groupoid if and only if it is a commutative regular semigroup.

Proof. If S is a T2CA-NET-groupoid. By Corollary 4 and Proposition 3 (2), we know that S is a commutative semigroup. By Definition 8, there are:

$$k*anti(k)*k = neut(k)*k = k.\ (\forall k\in S)$$

Therefore, element k is a regular element and S is a commutative regular semigroup.

Next, if S is a commutative regular semigroup. Applying Proposition 3 (1), we get S is a T2CA-groupoid. $\forall k\in S$, $\exists w\in S$ we have

$$k*(w*k) = k.$$

Also,

$$(w*k)*k = (w*k)*[k*(w*k)] = [(w*k)*(w*k)]*k = [(k*(w*k))*w]*k = (k*w)*k = k.$$

Therefore, there exists $(w*k)\in S$, such that $k*(w*k) = (w*k)*k = k$.

Moreover, since

$$w*k = w*[k*(w*k)] = [(w*k)*w]*k = k*[(w*k)*w].$$

Then,

$$\begin{aligned}&[(w*k)*w]*k\\ &= [(w*k)*w]*[k*(w*k)] = [(w*k)*((w*k)*w)]*k\\ &= [(w*k)*(w*(w*k))]*k = [((w*k)*(w*k))*w]*k\\ &= [(w * k) * w] * k && \text{(by } (w*k)*(w*k) = (w*k))\\ &= w*[k*(w*k)] = w*k.\end{aligned}$$

Thus, there exists $[(w*k)*w]\in S$, such that $k*[(w*k)*w] = [(w*k)*w]*k = w*k$. Then S is a T2CA-NET-groupoid. □

Example 8. T2CA-NET-groupoid of order 5, given in Table 8, and

$$neut(a)=a,\ \{anti(a)\} = \{a, w, r, t\};\ neut(q) = a,\ anti(q) = q;$$

$$neut(w) = r,\ anti(w) = w;\ neut(r) = r,\ anti(r) = r;\ neut(t) = t,\ anti(t) = t.$$

Also $(S, *)$ is a regular semigroup, due to the fact that $a = a*a*a$, $q = q*q*q$, $w = w*w*w$, $r = r*r*r$, $t = t*t*t$. Obviously, $(S, *)$ is a commutative.

Table 8. Cayley table on $S = \{a, q, w, r, t\}$.

*	a	q	w	r	t
a	a	q	a	a	a
q	q	a	q	q	q
w	a	q	r	w	a
r	a	q	w	r	a
t	a	q	a	a	t

Definition 9. Let $(S, *)$ be a T2CA-groupoid. (1) If $\forall\ k\in S$, $\exists\ s, t\in S$ such that $k*s = k$, and $t*k = s$. Then, S is called a T2CA-(r, l)-NET-groupoid.

(2) If $\forall\ k\in S$, $\exists\ s, t\in S$ such that $k*s = k$, and $k*t = s$. Then, S is called a T2CA-(r, r)-NET-groupoid.

(3) If $\forall\ k\in S$, $\exists\ s, t\in S$ such that $s*k = k$, and $k*t = s$. Then, S is called a T2CA-(l, r)-NET-groupoid.

(4) If $\forall\ k\in S$, $\exists\ s, t\in S$ such that $s*k = k$, and $t*k = s$. Then, S is called a T2CA-(l, l)-NET-groupoid.

Theorem 14. Suppose $(S, *)$ is a groupoid, then S is a T2CA- (r, l)-NET-groupoid if and only if it is a commutative regular semigroup.

Proof. If S is a T2CA-(r, l)-NET-groupoid. $\forall\ k\in S$, by Definition 8, Definition 9 (1) there are:

$$k*neut(k) = k,\ anti(k)*k = neut(k).$$

Moreover, we have

$$\begin{aligned}k*anti(k) &= [k*neut(k)]*anti(k) = [anti(k)*k]*neut(k) = neut(k)*neut(k),\\ neut(k)*k &= (anti(k)*k)*k = k*(k*anti(k)) = k*[(k*neut(k))*anti(k)]\\ &= (anti(k)*k)*(k*neut(k)) = neut(k)*(k*neut(k)) = [neut(k)*neut(k)]*k\\ &= (k*anti(k))*k && \text{(By } k*anti(k) = neut(k)*neut(k))\\ &= anti(k)*(k*k) = anti(k)*[(k*neut(k))*k] = (k*anti(k))*(k*neut(k))\\ &= (anti(k)*k)*(neut(k)*k) && \text{(Using Proposition 4 (1))}\\ &= neut(k)*(neut(k)*k) = (k*neut(k))*neut(k)\\ &= k*neut(k) = k.\end{aligned}$$

Thus, $neut(k)*k = k*neut(k) = k$.

Further, we have

$$\begin{aligned}k*anti(k) &= neut(k)*neut(k) = (anti(k)*k)*(anti(k)*k)\\ &= (k*anti(k))*(k*anti(k)) && \text{(Using Proposition 4 (1))}\end{aligned}$$

$$= anti(k)*[(k*anti(k))*k] = anti(k)*[anti(k)*(k*k)]$$
$$= anti(k)*[neut(k)*k] \qquad \text{(By } anti(k)*(k*k) = neut(k)*k\text{)}$$
$$= [k*anti(k)]*neut(k) = anti(k)*[neut(k)*k]$$
$$= anti(k)*k = neut(k).$$

Thus, $anti(k)*k = k*anti(k) = neut(k)$.

Therefore, we prove that S is a T2CA-NET-groupoid. By Theorem 13, we get that T2CA-(r, l)-NET-groupoid is equivalent to commutative regular semigroup. □

Theorem 15. Suppose (S, *) is a groupoid, then S is a T2CA- (r, r)-NET-groupoid if and only if it is a commutative regular semigroup.

Proof. If S is a T2CA-(r, r)-NET-groupoid. $\forall k\in S$, by Definition 8, Definition 9 (2) there are:

$$k * neut(k) = k,\ k * anti(k) = neut(k).$$

Moreover, we have

$$neut(k)*k = [k*anti(k)]*k = anti(k)*(k*k) = anti(k)*[k*(k*neut(k))]$$
$$= [(k*neut(k))*anti(k)]*k = [neut(k)*(anti(k)*k)]*k$$
$$= (anti(k)*k)*(k*neut(k)) = (anti(k)*k)*k = k*(k*anti(k))$$
$$= k*neut(k) = k.$$

Thus, $neut(k)*k = k*neut(k) = k$.

Further, we have

$$anti(k)*k = anti(k)*(neut(k)*k) = anti(k)*[neut(k)*(k*neut(k))]$$
$$= anti(k)*[(neut(k)*neut(k))*k] = (k*anti(k))*[neut(k)*neut(k)]$$
$$= neut(k)*[neut(k)*neut(k)] = neut(k)*[(k*anti(k))*neut(k)]$$
$$= neut(k)*[anti(k)*(neut(k)*k)] = neut(k)*[anti(k)*k]$$
$$= [k*neut(k)]*anti(k)$$
$$= k*anti(k) = neut(k).$$

That is, $anti(k)*k = k*anti(k) = neut(k)$.

Therefore, we prove that S is a T2CA-NET-groupoid. By Theorem 13, we get that T2CA-(r, r)-NET-groupoid is equivalent to commutative regular semigroup. □

Theorem 16. Suppose (S, *) is a groupoid, then S is a T2CA-(l, r)-NET-groupoid if and only if it is a commutative regular semigroup.

Proof. If S is a T2CA-(l, r)-NET-groupoid. $\forall k\in S$, by Definition 8, Definition 9(3) we have

$$neut(k)*k = k,\ k*anti(k) = neut(k).$$

Moreover,

$$anti(k)*k = anti(k)*(neut(k)*k) = (k*anti(k))*neut(k) = neut(k)*neut(k)$$
$$= neut(k)*[k*anti(k)] = [anti(k)*neut(k)]*k = [anti(k)*(k*anti(k))]*k$$
$$= [(anti(k)*anti(k))*k]*k = k*[k*(anti(k)*anti(k))]$$
$$= k*[(anti(k)*k)*anti(k)] = [anti(k)*k]*[anti(k)*k].$$

$$neut(k)*neut(k) = [neut(k)*neut(k)]*[neut(k)*neut(k)]$$
$$= [anti(k)*k]*[neut(k)*neut(k)] \qquad \text{(By } anti(k)*k = neut(k)*neut(k)\text{)}$$
$$= k*[(neut(k)*neut(k))*anti(k)] = k*[neut(k)*(anti(k)*neut(k))]$$
$$= [(anti(k)*neut(k))*k]*neut(k) = [neut(k)*(k*anti(k))]*neut(k)$$
$$= [neut(k)*neut(k)]*neut(k).$$

Then,

$$k*neut(k) = k*(k*anti(k)) = [anti(k)*k]*k$$
$$= [neut(k)*neut(k)]*k \qquad \text{(By } anti(k)*k = neut(k)*neut(k)\text{)}$$
$$= [(neut(k)*neut(k))*neut(k)]*k$$
$$\text{(By } [neut(k)*neut(k)]*neut(k)=neut(k)*neut(k)\text{)}$$
$$= [neut(k)*(neut(k)*neut(k))]*k = [neut(k)*neut(k)]*[k*neut(k)]$$
$$= neut(k)*[(k*neut(k))*neut(k)] = neut(k)*[neut(k)*(neut(k)*k)]$$

$= neut(k)*[neut(k)*k] = neut(k)*k = k.$

Thus, $neut(k)*k = k*neut(k) = k$.

Further, we have

$[neut(k)*neut(k)]*neut(k)= neut(k)*[neut(k)*neut(k)]$

$= neut(k)*[anti(k)*k]$ (By $anti(k)*k = neut(k)*neut(k)$)

$= [k*neut(k)]*anti(k)$

$= k*anti(k) = neut(k).$

Thus, $[neut(k)*neut(k)]*neut(k) = neut(k)*neut(k) = anti(k)*k = neut(k) = k*anti(k)$.

Therefore, we prove that S is a T2CA-NET-groupoid. By Theorem 13, we get that T2CA-(l, r) -NET-groupoid is equivalent to commutative regular semigroup. □

Theorem 17. Suppose (S, *) is a groupoid, then S is a T2CA-(l, l)-NET-groupoid if and only if it is a commutative regular semigroup.

Proof. If S is a T2CA-(l, l)-NET-groupoid. $\forall k\in S$, by Definition 8, Definition 9 (4) we have $neut(k)*k = k$, $anti(k)*k = neut(k)$. Moreover,

$k*neut(k) = k*[anti(k)*k]$

$= (k*k)*anti(k) = [(neut(k)*k)*k]*anti(k)$

$= [(k*(k*neut(k)))]*anti(k) = [k*neut(k)]*[anti(k)*k]$

$= [k*neut(k)]*neut(k) = neut(k)*[neut(k)*k]$

$= neut(k)*k = k.$

Thus, $k*neut(k) = neut(k)*k = k$.

Further, we have

$k*anti(k)$

$= [k*neut(k)]*anti(k) = neut(k)*[anti(k)*k] = neut(k)*neut(k)$

$= [anti(k)*k]*neut(k) = k*[neut(k)*anti(k)] = k*[(anti(k)*k)*anti(k)]$

$= k*[k*(anti(k)*anti(k))] = [(anti(k)*anti(k))*k]*k = [anti(k)*(k*anti(k))]*k$

$= [k*anti(k)]*[k*anti(k)] = anti(k)*[(k*anti(k))*k] = anti(k)*[anti(k)*(k*k)]$

$= [(k*k)*anti(k)]*anti(k) = [k*(anti(k)*k)]*anti(k) = [k*neut(k)]*anti(k)$

$= [neut(k)*k]*anti(k) = k*[anti(k)*neut(k)] = k*[anti(k)*(anti(k)*k)]$

$= k*[(k*anti(k))*anti(k)] = [anti(k)*k]*[k*anti(k)] = neut(k)*(k*anti(k))$

$= neut(k)*(neut(k)*neut(k))$ (By $neut(k)*neut(k)= k*anti(k)$)

$= [neut(k)*neut(k)]*neut(k))$

$= [k*anti(k)]*neut(k)$ (By $neut(k)*neut(k)= k*anti(k)$)

$= anti(k)*[neut(k)*k] = anti(k)*k = neut(k).$

Thus, $anti(k)*k = k*anti(k) = neut(k)$.

Therefore, we prove that S is a T2CA-NET-groupoid. By Theorem 13, we get that T2CA-(l, l)-NET-groupoid is equivalent to commutative regular semigroup. □

Example 9. T2CA-**(**r, l)**-**NET-groupoid of order 4, given in Table 9, and

$neut_{(r,l)}(c) = c$, $\{anti_{(r,l)}(c)\} = \{c, v, b, n\}$; $neut_{(r,l)}(v) = n$, $\{anti_{(r,l)}(v)\} = v$;

$neut_{(r,l)}(b) = b$, $anti_{(r,l)}(b) = b$; $neut_{(r,l)}(n) = n$, $anti_{(r,l)}(n) = n$.

It is easy to verify that (S, *) is also a T2CA-(r, r)-NET-groupoid, T2CA-(l, r)-NET-groupoid, T2CA-(l, l)-NET-groupoid. Moreover, (S, *) is a regular semigroup**,** due to the fact that $c = c*c*c$, $v = v * v * v$, $b = b * b * b$, $n = n* n * n$. Obviously, (S, *) is a commutative.

Table 9. Cayley table on $S = \{c, v, b, n\}$.

*	*c*	*v*	*b*	*n*
c	*c*	*c*	*c*	*c*
v	*c*	*n*	*c*	*v*
b	*c*	*c*	*b*	*c*
n	*c*	*v*	*c*	*n*

6. Quasi Neutrosophic Extended Triplet (QNET) Groupoids and T2CA-QNET-Groupoids

Definition 10. Let *(S, *)* be a groupoid. If for any $x \in S$, there exists $y, z \in S$ such that

$$(x*y = x, \text{ or } y*x = x), \text{ and } (z*x = y \text{ or } x*z = y).$$

then *(S, *)* is called a quasi neutrosophic extended triplet groupoid (shortly, QNET-groupoid). Suppose *(S, *)* is a semigroup and QNET-groupoid, then *(S, *)* is called a quasi neutrosophic triplet group (shortly, QNETG). Suppose *(S, *)* is a T2CA-groupoid and QNET-groupoid, then *(S, *)* is called a T2CA-QNET-groupoid.

Let *(S, *)* be a QNET-groupoid and $x \in S$. We introduce the following concepts:

(1) If $\exists y, z \in S$, *s.t.* $x * y = x$ and $x * z = y$, then x is called an QNET-element with (r-r)- property;
(2) If $\exists y, z \in S$, *s.t.* $x * y = x$ and $z * x = y$, then x is called an QNET-element with (r-l)- property;
(3) If $\exists y, z \in S$, *s.t.* $y * x = x$ and $z * x = y$, then x is called an QNET-element with (l-l)- property;
(4) If $\exists y, z \in S$, *s.t.* $y * x = x$ and $x * z = y$, then x is called an QNET-element with (l-r)- property;
(5) If $\exists y, z \in S$, *s.t.* $x * y = y * x = x$ and $z* x=y$, then x is called an QNET-element with (lr-l)-property;
(6) If $\exists y, z \in S$, *s.t.* $x * y = y * x = x$ and $x* z= y$, then x is called an QNET-element with (lr-r)-property;
(7) If $\exists y, z \in S$, *s.t.* $y * x = x$ and $x * z= z* x = y$, then x is called an QNET-element with (l-lr)-property;
(8) If $\exists y, z \in S$, *s.t.* $x * y=x$ and $x * z = z * x = y$, then x is called an QNET-element with (r-lr)-property;
(9) If $\exists y, z \in S$, *s.t.* $x * y = y * x = x$ and $x * z = z * x = y$, then x is called an QNET-element with (lr-lr)-property.

Easy to verify: (i) if x is an QNET-element with (r-lr)-property, then x is an QNET-element with (r-r)-property and (r-l)-property; if x is an QNET-element with (lr-r)-property, then x is an QNET-element with (l-r)-property and (r-r)-property; and soon; (ii) if * is commutative, then the above properties coincide.

Example 10. Denote S = {1, 2, 3, 4}, define the operation * on S in Table 10. Then *(S, *)* is QNET-groupoid, and 1 is an QNET-element with (lr-lr)-property; 2 is an QNET-element with (lr-r)-property; 3 is an QNET-element with (r-r)-property; and 4 is an QNET-element with (l-lr)-property. Obviously, (*S*, *) is not a NET-groupoid.

Table 10. The operation ***** on *S*

*	1	2	3	4
1	1	1	1	4
2	3	1	2	3
3	3	2	4	1
4	1	4	2	1

Example 11. Denote S = {1, 2, 3, 4, 5}, define the operation * on S in Table 11. Then *(S, *)* is QNETG, and 1 is an QNET-element with (lr-lr)-property; 2 is an QNET-element with (lr-lr)-property; 3 is an QNET-element with (lr-lr)-property; 4 is an QNET-element with (lr-r)-property; and 5 is an QNET-element with (lr-r)-property. Obviously, (*S*, *) is not a NETG. Moreover, since $5 * (5 * 4) = 5 \neq 1 = (4 * 5) * 5$, (*S*, *) is not a T2CA-groupoid.

Table 11. The operation ***** on *S*

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	4	1
3	1	1	3	1	5
4	1	4	1	1	2
5	1	1	5	3	1

Theorem 18. Suppose $(S, *)$ is a groupoid, then S is a T2CA-QNET-groupoid if and only if it is a T2CA-NET-groupoid.

Proof. Let $(S, *)$ be a T2CA-QNET-groupoid. In particular, we consider the local unit element of each element. By Definition 10, we know that for any $a, b \in S$, there are four cases of their local unit.

Case 1: There exists $y, z \in S$, such that $a*y = a$, $b*z = b$. Then a is an QNET-element with (r-l, or r-r, or r-lr)-property, b is an QNET-element with (r-l, or r-r, or r-lr)-property. We have

$$
\begin{aligned}
a*b &= (a*y)*b = y*(b*a) = y*[b*(a*y)] = y*[(y*b)*a] \\
&= (a*y)*(y*b) = a*(y*b) = (b*a)*y = [(b*z)*a]*y \\
&= [z*(a*b)]*y = (a*b)*(y*z) \\
&= (b*a)*(z*y) && \text{(Applying Proposition 4 (1))} \\
&= [y*(b*a)]*z = [(a*y)*b]*z = (a*b)*z = b*(z*a) \\
&= z*[(z*a)*b] = z*[a*(b*z)] = z*(a*b) = (b*z)*a \\
&= b*a.
\end{aligned}
$$

Case 2: There exists $y, z \in S$, such that $y*a = a$, $z*b = b$. Then a is an QNET-element with (l-l, or l-r, or l-lr)-property, b is an QNET-element with (l-l, or l-r, or l-lr)-property. According to case 1, we can similarly get

$$a*b = (y*a)*(z*b) = (a*y)*(b*z) = b*a.$$

Case 3: There exists $y, z \in S$, such that $y*a = a$, $b*z = b$. Then a is an QNET-element with (l-l, or l-r, or l-lr)-property, b is an QNET-element with (r-l, or r-r, or r-lr)-property. We have

$$
\begin{aligned}
z*a &= z*(y*a) = z*[y*(y*a)] = [(y*a)*z]*y \\
&= [a*(z*y)]*y = (z*y)*(y*a) \\
&= (z*y)*a;
\end{aligned}
$$

$$
\begin{aligned}
b*a &= b*(y*a) = (a*b)*y = [(y*a)*b]*y = [a*(b*y)]*y = (b*y)*(y*a) \\
&= (b*y)*a = y*(a* b) && \text{(By } a*b = a*(b*y)\text{)} \\
&= y*[a*(b*y)] = [(b*y)*y]*a = [y*(y*b)]*a = (y*b)*(a*y) \\
&= (b*y)*(y*a) && \text{(Applying Proposition 4 (1))} \\
&= (b*y)*a \\
&= y*(a*b);
\end{aligned}
$$

$$
\begin{aligned}
a*b &= a*(b*z) = (z*a)*b = [(z*y)*a]*b && \text{(By } z*a = (z*y)*a\text{)} \\
&= a*[b*(z*y)] = a*[(y*b)*z] \\
&= (z*a)*(y*b) = (a*z)*(b*y) && \text{(Applying Proposition 4 (1))} \\
&= z*[(b*y)*a] = z*[y*(a*b)] \\
&= z*(b*a). && \text{(By } y*(a*b) = b*a\text{)}
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
b &= b*z = (b*z)*z = z*(z*b) = z*[z*(b*z)] = [(b*z)*z]*z = [z*(z*b)]*z \\
&= (z*b)*(z*z) = (b*z)*(z*z) && \text{(Applying Proposition 4 (1))} \\
&= b*(z*z).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
a*b &= a*(b*z) = a*[b*(z*z)] && \text{(By } b*z = b*(z*z)\text{)} \\
&= [(z*z)*a]*b = [z*(a*z)]*b= (a*z)*(b*z) \\
&= (z*a)*(z*b) && \text{(Applying Proposition 4 (1))} \\
&= [b*(z*a)]*z = [(a*b)*z]*z \\
&= (b*a)*z = [(b*a)*z]*z= z*[z*(b*a)] \\
&= z*(a*b) && \text{(By } a*b = z*(b*a)\text{)} \\
&= (b*z)*a = b*a.
\end{aligned}
$$

Case 4: There exists $y, z \in S$, such that $a*y = a$, $z*b =b$. Then a is an QNET-element with (r-l, or r-r, or r-lr)-property, b is an QNET-element with (l-l, or l-r, or l-lr)-property. According to case 3, we can similarly get $a*b = (a*y)*(z*b) = (y*a)*(b*z) = b*a$.

From Case 1, Case2, Case 3, and Case 4, we know that S is a commutative T2CA-QNET-groupoid. Then, for any $x \in S$, there exists y, $z \in S$ such that $x*y = x$, $y*x = x$, and $z*x = y$, $x*z = y$. Therefore, we prove that S is a T2CA-NET-groupoid.

Conversely, it is obvious. □

Corollary 5. Assume (S, *) is a T2CA-QNET-groupoid, then (S, *) is a QNETG.

Proof. Assume that S is a T2CA-QNET-groupoid. By Theorem 18 and 13, we get that S is a commutative regular semigroup. According to the Definition 10, we get that S is a QNETG. □

The inverse of Corollary 5 is not true, see Example 11.

Corollary 6. Let (S, *) be a T2CA-groupoid. Then, the following statements are equivalent:
(i) S is a T2CA-QNET-groupoid;
(ii) S is a T2CA-NET-groupoid;
(iii) S is a CA-NET-groupoid;
(iv) S is a commutative regular semigroup.

Proof. (i) Þ (ii). Suppose that S is a T2CA-QNET-groupoid. Applying Theorem 18, we get that S is a T2CA-NET-groupoid.

(ii) Þ (iii). Suppose that S is a T2CA-NET-groupoid. Applying Theorem 13, we get that S is a commutative regular semigroup. Then by Theorem 2 (1) and (6), we get S is a CA-NET-groupoid.

(iii) Þ (iv). Suppose that S is a CA-NET-groupoid. Applying Theorem 2 (1) and (6), we get that S is a commutative regular semigroup.

(iv) Þ (i). Suppose that S is a commutative regular semigroup. Applying Theorem 13, we get S is a T2CA-NET-groupoid. Then by Theorem 18, S is a T2CA-QNET-groupoid. □

7. Conclusions

In the paper, we introduced the new concepts of T2CA-groupoid, T2CA-NET-groupoid, and QNET-groupoid for the first time. We precisely discussed some fundamental characteristics of T2CA-groupoids and T2CA-NET-groupoids, then a decomposition theorem of T2CA-NET-groupoid is proved (see Theorem 8), and the relationship between T2CA-NET-groupoids and commutative regular semigroups is strictly proved. Furthermore, we investigated relationships among T2CA-QNET-groupoid, T2CA-NET-groupoid, CA-NET-groupoid and commutative regular semigroup**.** The results show that T2CA-groupoids, as a non-associative algebraic structure, are typically representative and closely related to a variety of algebraic structures.

For future research directions, we will discuss the integration of the related topics (such as algebraic systems related fuzzy logics and non-associative groupoids, see [29-34]).

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