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New Types of Neutrosophic Crisp Closed Sets

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Abstract: The neutrosophic sets were known since 1999, and because of their wide applications and their great flexibility to solve the problems, we used these the concepts to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namly [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respactvely, we stady their properties in details and join it with topological concepts. Finally we used [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] to introduce of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp extrior and neutrosophic crisp boundary which are fundamental for further reserch on neutrosophic crisp topology and will setrengthen the foundations of theory of neutrosophic topological spaces.

Keywords: Neutrosophic crisp set, Neutrosophic crisp topology, Neutrosophic crisp closed set,

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set [1] which is the generalization of the class sets, conventional fuzz set [2] and intuitionistic set fuzzy [3]. After Smarandache, neutrosophic sets have been successfully applied to many fields such as; topology, control theory, databases, medical diagnosis problem, decision making problem and so on, [4-37] .

A.A. Salama, et, al.[38] proposed a new mathematical model called " Neutrosophic crisp sets and Neutrosophic crisp topological spaces " .

The idea of "Gem-Set", which is a characterization of the concept of closure is introduced by AL-Nafee ,Al-Swidi [39] . After AL-Nafee, the idea of "Gem-Set has been successfully using to many topological concepts such as; interior, exterior, boundary ,separation axioms, continuous functions , bitopological spaces, compactness, soft topological spaces, and so on, [40,41,42,43,44,45,46,47,48].

The idea of "controlling soft Gem-Set" and join it with topological concepts in soft topological space is introduced by [49]. The concept of the soft Turing point and used it with separation axioms in soft topological space is introduced by [50,51].

The goal of this research is to combine the concept of "Gem-Set" and Turing point with neutrosophic crisp set to define a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space, namly [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respactvely, we stady their properties in details and we also use it to introduce the some of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure, neutrosophic crisp interior, neutrosophic crisp extrior and neutrosophic crisp boundary which are fundamental for further reserch on neutrosophic crisp topology and will setrengthen the foundations of theory of neutrosophic topological spaces.

The paper is structured as follows; In section 2, we first recall the necessary background on neutrosophic and neutrosophic crisp points [NCP_N for short]. In section 3, a neutrosophic crisp Turing points properties are introduced with their properties. In section 4, the concept of neutrosophic crisp Gem sets are introduced and studied their properties.

Throughout this paper ,NCTS means a neutrosophic crisp topological space , also we write (H) by H (for short),the collection of all neutrosophic crisp sets on H will be denoted by N(H) .

2. Preliminaries

2.1. **Definition** [52]

Let H be a non-empty fixed set, a neutrosophic crisp set (for short NCS) D is an object having the form $D = <D_1,D_2,D_3>$ where D_1,D_2 and D_3 are subsets of H .

We will exhibit the basic neutrosophic operations defnitions (union, intersection and complement) . Since there are different definitions of neutrosophic operations , we will organize the existing definitions into two types in each type these operations will be consistent and functional . In this work we will use one Type of neutrosophic crisp sets operations .

2.2. **Definition** [52]

A neutrosophic crisp topology (NCTS) on anon-empty set H is a family T of neutrosophic crisp susets in H satisfying the following conditions ;

$\emptyset_N, H_N \in T$.

$C \cap D \in T$, for $C,D \in T$

The union of any number of set in T belongs to T .

The pair (H,T) is said to be a neutrosophic crisp topological space(NCTS) in H . Moreover the elements in T are said to be neutrosophic crisp open sets. A neutrosophic crisp set F is closed iff its complement (F^C) is an open neutrosophic crisp set .

2.3. **Definition** [52]

Let NI be a non-null collection of neutrosophic crisp sets over a universe H . Then NI is called neutrosophic crisp ideal on H if ;

- $C \in NI$ and $D \in NI$ then $C \cup D \in NI$.
- $C \in NI$ and $D \subseteq C$ then $D \in NI$.

2.4. **Definition** [52]

Let (H,) be NCTS ,A be a neutrosophic crisp set then: The intersection of any neutrosophic crisp closed sets contained A is called neutrosophic crisp clusuer of A (for short NC-CL(A)) .

2.5. **Definition** [52]

((neutrosophic crisp sets operations of Type.I))

Let H be a non-empty set and $C = < C_1, C_2, C_3 >$, $D = < D_1, D_2, D_3 >$ be two neutrosophic crisp sets, where D_1, C_1,D_2,C_2 and D_3 C_1 are subsets of H ,such that $(D_1 \cap D_2)=\emptyset$, $(D_1 \cap D_3)=\emptyset$, $(D_2 \cap D_3)=\emptyset$, $(C_1 \cap C_2)=\emptyset$, $(C_1 \cap C_3)=\emptyset$, $(C_2 \cap C_3)=\emptyset$ then:

- $\emptyset_N = <\emptyset,\emptyset,H>$ (Neutrosophic empty set) .
- $H_N =<H,\emptyset,\emptyset>$ (Neutrosophic universal set) .
- $C \cap D = [C_1 \cap D_1] ,[C_2 \cap D_2]$ and $[C_3 \cup D_3]$.
- $C \cup D = [C_1 \cup D_1] ,[C_2 \cup D_2]$ and $[C_3 \cap D_3]$.
- $C \subseteq D \Leftrightarrow C_1 \subseteq D_1 ,C_2 \subseteq D_2$ and $D_3 \subseteq C_3$.

- The complement of a NCS (D) may be defined as: D = < D_3 , D_2 , D_1 > .
- C = D ⇔ C ⊆ D , D ⊆ C .

2.6. **Definition** [53]

((neutrosophic crisp sets operations of Type.2))

Let H be a non-empty set and C = < C_1, C_2, C_3 > , D = < D_1, D_2, D_3 > be two neutrosophic crisp sets, where D_1, C_1, D_2 ,C_2 and D_3 ,C_1 are subsets of H then:

- $\emptyset_N$ = <∅, ∅, ∅> (Neutrosophic empty set) .
- H_N =<H,H,H> (Neutrosophic universal set) .
- C ∩ D = [C_1 ∩ D_1], [C_2 ∩ D_2] and [C_3 ∩ D_3] .
- C ∪ D = [C_1 ∪ D_1], [C_2 ∪ D_2] and [C_3 ∪ D_3] .
- C ⊆ D ⇔ C_1 ⊆ D_1 , C_2 ⊆ D_2 and C_3 ⊆ D_3 .
- The complement of a NCS (D) may be defined as: D^C = <D_1^c, D_2^c, D_3^c > .
- C = D ⇔ C ⊆ D , D ⊆ C".

2.7. **Definition** [53]

For all a,b,c ∈ H .Then the neutrosophic crisp points related to a,b,c are defined as follows ;

- a_{N_1} = <{a},∅,∅> on H .
- b_{N_2} = <∅,{b},∅> on H .
- c_{N_3} =<∅,∅,{c}> on H .

(The set of all neutrosophic crisp points ($a_{N_1}, b_{N_2}, c_{N_3}$) is denoted by NCPN) .

3. **Neutrosophic crisp turing point**

In this work, we will use Type.2 of neutrosophic crisp sets operations , this was necessary to homogeneous suitable results for the upgrade of this research .

3.1. **Definition**

Let (H, T) be NCTS , P ∈ NCPN in H, we define a neutrosophic crisp ideal NI with respect to a neutrosophic crisp point P, as follows :

$$NI(P) = \{D \in T : P \in (D)^C\}$$

3.2. **Definition**

Let (H, T) be NCTS, P ∈ NCPN in (H, T), Y⊆ H, we define a neutrosophic crisp ideal $^{Y}NI(P)$ respect to subspace (Y,T_Y), as follows:

$$^{Y}NI(P) = \{D \in T_Y : P \in (H \setminus D)\} .$$

3.3. **Remark**

Let (H, T) be NCTS, Y⊆H, for each D≠ $\emptyset_N$ and P ∈ NCPN in Y , then ;

$$^{Y}NI(P) = \{ D \in T_Y : P \in (H\setminus D)\} = \{D \in T_Y : P \in (Y\setminus D) \} .$$

Proof

$^{Y}NI(P) = \{D \in T_Y : P \in (H\setminus D)\} = \{ D \in T_Y : P \notin D, \text{for each } P \in Y\} = \{ D \in T_Y : P \in (Y\setminus D), \text{ for each } P \in Y\}$.

3.4. **Remark**

Let (H, T) be NCTS ,Y⊆ H, for each D≠ $\emptyset_N$ and P ∈ NCPN in H,then ;

$$^{Y}NI(P) = \{ D \in T_Y : P \in (H\setminus D)\} = \{D \cap Y : \text{for each } D \neq \emptyset_N \in NI(P)\}.$$

3.5. **Example**

Let (H, T) be NCTS , such that H={1} ,

T={ $\emptyset_N$,H_N, A , B ,C ,D ,E ,F,G } , P_1 =< ∅.{1},∅ > , such that;

A =< {1}, ∅, ∅ >, B =< ∅, {1}, ∅ >, C =< {1}, {1}, ∅ > , D =< {1}, ∅, {1} > ,

$E =< \emptyset, \{1\}, \{1\} >$, $F =< \emptyset, \emptyset, \{1\} >$, $G =< \{1\}, \{1\}, \{1\} >$.

Then, $NI(P_1) =\{\emptyset_N$, A,D,F }.

3.6. **Definition**

Let (H, T) be NCTS , P ∈ NCPN in H and NI be a neutrosophic crisp ideal on (H, T), we say that p is a neutrosophic crisp turing point of NI if $D^c \in NI$، for each $D \in T_P$,T_P is collection of all neutrosophic crisp open set of neutrosophic crisp point p.

3.7. **Remark**

Let (H, T) be NCTS, P ∈ NCPN in H and $NI(P) =\{D \in T: P \in (D)^C\}$ be aneutrosophic crisp ideal on (H, T) .

Then, p is a neutrosophic crisp turing point of NI(P) .

3.8. **Example**

Let (H, T) be NCTS , such that H={1},

T={ $\emptyset_N$, H_N, A , B ,C ,D ,E ,F,G }, $P_1 = < \emptyset, \{1\}, \emptyset >$, $P_2 = < \{1\}, \emptyset, \emptyset >$, such that;

$A = < \{1\}, \emptyset, \emptyset >$, $B = < \emptyset, \{1\}, \emptyset >$, $C = < \{1\}, \{1\}, \emptyset >$, $D = < \{1\}, \emptyset, \{1\} >$,

$E = < \emptyset, \{1\}, \{1\} >$, $F = < \emptyset, \emptyset, \{1\} >$, $G = < \{1\}, \{1\}, \{1\} >$.

Then, P_1 is a neutrosophic crisp turing point of neutrosophic crisp ideal $NI(P_1)$,but not P_2 .

3.9. **Theorem**

Let (H, T) be NCTS, $a_{N_1} \neq b_{N_1} \in$ NCPN in H, then, $< \{b\}, \emptyset, \emptyset >$ is a neutrosophic crisp closed set if and only if a_{N_1} is not a aneutrosophic crisp turing point of $NI(b_{N_1})$.

Proof

Let $a_{N_1} \neq b_{N_1} \in$ NCPN in H. Assume that$< \{b\}, \emptyset, \emptyset >$ is a neutrosophic crisp closed set ,so that $< \{b\}, \emptyset, \emptyset > = cl\ (< \{b\} , \emptyset, \emptyset >)$. But $a_{N_1} \neq b_{N_1}$ get that $a_{N_1} \notin cl\ (< \{b\} , \emptyset, \emptyset >)$. Therefore, there exists a neutrosophic crisp open set U such that, $a_{N_1} \in U$, $U \cap < \{b\}, \emptyset, \emptyset > =\emptyset_N$. So that $a_{N_1} \in U$, $U^c \notin NI(b_{N_1})$,because if $U^c \in NI(b_{N_1})$, then$< \{b\}, \emptyset, \emptyset > \in U$,that means $U \cap < \{b\} , \emptyset, \emptyset > \neq \emptyset_N$,this a contradiction!. Hence $a_{N_1}$is not a neutrosophic crisp turing point of $NI(b_{N_1})$.

Conversely,

Let $a_{N_1} \neq b_{N_1} \in$ NCPN in H. Since a_{N_1} is not a neutrosophic crisp turing point of $NI(b_{N_1})$, then there exists a neutrosophic crisp open set U such that, $a_{N_1} \in U$, $U^c \notin NI(b_{N_1})$, so $< \{b\}, \emptyset, \emptyset > \notin U$.Thus $a_{N_1} \in U$, $U \cap < \{b\}, \emptyset, \emptyset > =\emptyset_N$ implies $a_{N_1} \notin cl(< \{b\}, \emptyset, \emptyset >)$.

Hence $< \{b\}, \emptyset, \emptyset >= cl(< \{b\}, \emptyset, \emptyset >)$, thus $< \{b\}, \emptyset, \emptyset >$ is a neutrosophic crisp closed set in H.

Proof by the same proof of 2.10. Theorem .

4. **Neutrosophic crisp Gem set**

4.1. **Definition**

Let (H, T) be NCTS, P ∈ NCPN in H ,NI(P) be aneutrosophic crisp ideal on (H, T) and D ⊆(H, T), we defined the neutrosophic crisp set ND^{*P} with respect to space (H, T) as follows:

$ND^{*P} =\{P_1 \in$ NCPN in H ; $F \cap D \notin NI(P)$, for each $F \in T_{P_1}$,$T_{P_1}$is collection of all neutrosophic crisp open set of neutrosophic crisp point P_1.The neutrosophic crisp set ND^{*P} is called neutrosophic crisp Gem-Set .

4.2. **Example**

Let (H, T) be NCTS , such that H={1,2,3},

T ={ ∅_N, H_N, A , B ,C ,D ,E ,F,G }, $P = < \emptyset, \{1\} , \emptyset >$, $D = < \emptyset,\{1,3\} , \emptyset >$, such that;

$A = < \emptyset,\{1\} ,\emptyset >$, $B = < \emptyset,\{2\},\emptyset >$, $C = < \emptyset, \{3\} ,\emptyset >$, $D = < \emptyset,\{1,2\},\emptyset >$.

$E = <\emptyset,\{1,3\},\emptyset,\emptyset>$, $F = <\emptyset,\{2,3\},\emptyset>$, $G = <\emptyset,\{1,2,3\},\emptyset>$.

Then, $NI(P) = \{\emptyset, N, B, C, F\}$ and $ND^{*P} = <\{1\},\emptyset,\emptyset>$.

4.3. **Theorem**

Let (H,T) be NCTS, $P \in$ NCPN in H, and let D,C be subsets of (H,T). Then

1. $\emptyset_N{}^{*P} = \emptyset_N$
2. $H_N{}^{*P} = H_N$, whenever $NI(P) = \emptyset_N$.
3. $C \subseteq D \rightarrow NC^{*P} \subseteq ND^{*P}$.
4. For any points $P_1, P_2 \in$ NCPN in H, with $NI(P_2) \supseteq NI(P_1)$ then $ND^{*P_2} \subseteq ND^{*P_1}$.
5. $P \in D$ if and only if $P \in ND^{*P}$.
6. If $P \in D$, then $(ND^{*P})^{*P} = ND^{*P}$.
7. If $P_1 \in D$, $P_2 \in C$ with $P_1 \neq P_2$, $D \cap C = \emptyset_N$, then $ND^{*P_1} \cap NC^{*P_2} = \emptyset_N$.
8. If $a_{N_1}, b_{N_1} \in$ NCPN in Hwith $a_{N_1} \neq b_{N_1}$, then $b_{N_1} \in (a_{N_1})^c$ implies $a_{N_1} \notin (a_{N_1})^* b_{N_1}$ and $b_{N_1} \notin (b_{N_1})^* a_{N_1}$.

4.4 . **Remark**

The equality of theorem part (3),(4) does not necessarily hold as shown :

Let (H,T) be NCTS , such that $H=\{1,2\}$, $D = <\emptyset,\{2\},\emptyset>$, $C = <\emptyset,\{1\},\emptyset>$,
$T = \{\emptyset_N, H_N, A, B, G\}$, $P_1 = <\emptyset,\{2\},\emptyset>$, $P_2 = <\emptyset,\{1\},\emptyset>$,
$A = <\emptyset,\{1\},\emptyset>$, $B = <\emptyset,\{2\},\emptyset>$, $G = <\emptyset,\{1,2\},\emptyset>$,
Then, $NI(P_1) = \{\emptyset_N, A\}$, $NI(P_2) = \{\emptyset_N, B\}$ and $ND^{*P_1} = <\emptyset,\{2\},\emptyset>$, $ND^{*P_2} = \emptyset_N$, $NC^{*P_1} = \emptyset_N$

Note that,

1) $ND^{*P_2} \subseteq ND^{*P_1}$, but $NI(P_2) \not\supseteq NI(P_1)$.
2) $NC^{*P_1} \subseteq ND^{*P_1}$, but $C \nsubseteq D$.

4.5 .**Theorem**

Let (H,T) be NCTS, $P_1 \in$ NCPN in H and D,C be subsets of (H,T). Then $ND^{*P_1} \cup NC^{*P_1} = N(D \cup C)^{*P_1}$.

Proof

It is obviously known that $D \subset (D \cup C)$ and $C \subset (D \cup C)$, then from theorem 3.3 part(3) we get, $ND^{*P_1} \subset N(D \cup C)^{*P_1}$ and $ND^{*P_1} \subset N(A \cup C)^{*P_1}$, for any $P_1 \in$ NCPN in H. Hence

$$ND^{*P_1} \cup NC^{*P_1} \subset N(D \cup C)^{*P_1} \quad \text{----(1)}$$

For reverse inclusion, let $P_2 \notin ND^{*P_1}$. Then there exists neutrosophic crisp open set U containing p ,with $D \cap U \in NI(P_1)$. Similarly, if $P_2 \notin NC^{*P_1}$, then there exists neutrosophic crisp open set V containing P, with $C \cap V \in NI(P_1)$ .Then by hereditary property of neutrosophic crisp ideal, we get , $D \cap U \cap V \in NI(P_1)$ and $C \cap U \cap V \in NI(P_1)$. Again by the finite additivity condition of neutrosophic crisp ideal, we get $(D \cup C) \cap U \cap V \in NI(P_1)$. Hence $P_2 \notin N(D \cup C)^{*P_1}$. So,

$$N(D \cup C)^{*P_1} \subset ND^{*P_1} \cup NC^{*P_1} \quad \text{----(2).}$$

From (1) and (2) we get, $ND^{*P_1} \cup NC^{*P_1} = N(D \cup C)^{*P_1}$.

4.6 .**Theorem**

Let $(H.T)$ be NCTS, $P_1 \in$ NCPN in H and D,C be subsets of (H,T).Then $N(D \cap C)^{*P_1} \subset ND^{*P_1} \cap NC^{*P_1}$.

Proof

It is known that D∩C⊂D and D∩C⊂C, then from theorem part (3), $N(D \cap C)^{*P_1} \subset ND^{*P_1}$ and $N(D \cap C)^{*P_1} \subset NC^{*P_1}$.Hence $N(D \cap C)^{*P_1} \subset ND^{*P_1} \cap ND^{*P_1}$,for any $P_1 \in$ NCPN in H.

4.7 **.Theorem**

Let (H,T) be NCTS, $a_{N_1} \in$ NCPN in H, for each neutrosophic crisp open set U containinga_{N_1}, then $(a_{N_1})^{*}a_{N_1} \subseteq U$.

proof

Let $b_{N_1} \notin U$,so $a_{N_1} \neq b_{N_1}$,then we get that $U \cap (b_{N_1}) = \emptyset_N \in NI((a_{N_1})$.That means $(b_{N_1} \notin (a_{N_1})^{*}a_{N_1}$ .Thus$\{(a_{N_1})^{*}a_{N_1} \subseteq U$.

4.8 **.Theorem**

Let (H,T) be NCTS, $P_1 \in$ NCPN in H and D be subsets of (H,T).Then

$$D^{*P_1} = \begin{cases} \emptyset_N & \text{if } P_1 \notin D \\ cl(P_1) & \text{if } P_1 \in D \end{cases}$$

Proof

Case(1)

If $P_1 \notin D$,To prove $D^{*P_1} = \emptyset_N$ .Let $D^{*P_1} \neq \emptyset_N$, then there exists least one element. say $P_2 \in D^{*P_1}$ (by definition of D^{*P_1}), we have $C_{P_2} \cap D \notin NI(P_1)$. Hence$P_1 \in D \cap C_{P_2}$ So $P_1 \in D$ which contradiction!, then $D^{*P_1} = \emptyset_N$.

Case (2)

If $P_1 \in D$, to prove $D^{*P_1} = cl(P_1)$. Let $P_2 \in D^{*P_1}$ implies $P_1 \in D \cap V_{P_2}$ for each $V_{P_2} \in T_{P_2}$ implies that $P_1 \in V_{P_2}$ for each $V_{P_2} \in T_{P_2}$ it follows $P_2 \in cl(P_1)$ then $D^{*P_1} \subseteq cl(P_1)$ for each D be subsets of $(H.T)$. Let $P_2 \in cl(P_1)$ and $P_2 \notin D^{*P_1}$ then there exists neutrosophic crisp open set V_{P_2} containing P_2 such that $D \cap V_{P_2} \in NI(P_1)$, which implies that $P_1 \notin D \cap V_{P_2}$ then $P_1 \notin D$ or $P_1 \notin V_{P_2}$ which means that $P_1 \notin D$ or $P_2 \notin cl(P_1)$ which contradiction! in two case. Hence $P_2 \in D^{*P_1}$ implies that$cl(P_1) \subseteq D^{*P_1}$. Therefore, $D^{*P_1} = cl(P_1)$. if $P_1 \in D$.

4.9. **Definition**

Let (H,T) , (Y,δ) be NCTS. Then, the mapping $f:(H,T) \rightarrow (Y,\delta)$ is called NI*- map ,if and only if, for every subset D of (H,T), $P_1 \in$ NCPN in H, $f(D^{*P_1}) = (f(D))^{*f(P_1)}$ **.**

4.10. **Example**

Let (H,T) , (Y,δ) be NCTS, such that H={1,2,3}, Y={a,b,c},

T={ $\emptyset_N$, H_N, A , B }, δ={ $\emptyset_N$, Y_N, G }, such that .

A $< \{1\}, \emptyset, \emptyset >$, B= $< \{2,3\}, \emptyset, \emptyset >$, G $=< \{a\}, \emptyset, \emptyset >$.

Define f(2)=f(1)=c and f(3)=a, Put D={3} subset of (H,T) .

Then D^{*3} = B= $< \{2,3\}, \emptyset, \emptyset >$, so $f(D^{*3}) = (f(D))^{*f(3)=a} = (< \{a,c\}, \emptyset, \emptyset >)^{*}$**a** $=< \{a,b,c\}, \emptyset, \emptyset >$.

4.11. **Definition**

Let (H,T) , (Y,δ) be NCTS.Then, the mapping $f:(H,T) \rightarrow (Y,\delta)$ is called NI**-map if and only if, for every subset D of (Y,δ) ,p ∈ NCPN in Y, $f^{-1}(D^{*P}) = (f^{-1}(D))^{*f^{-1}(P)}$.

4.12. **Example**

Let (H,T) , (Y,δ) be NCTS, such that H={a,b,c}, Y={1,2,3}

T={ $\emptyset_N$, H_N, A , B }, δ={ $\emptyset_N$, Y_N, G }, such that .

$A = \langle \{a\}, \emptyset, \emptyset \rangle$, $B = \langle \{b, c\}, \emptyset, \emptyset \rangle$, $G = \langle \{1\}, \emptyset, \emptyset \rangle$.

Define f(b)=f(a)=3 and f(c)=1.Put D={3} subset of (Y, δ) .

Then $D^{*1} = B = \langle \{2,3\}, \emptyset, \emptyset \rangle$, so $f^{-1}(D^{*1}) = (f^{-1}(D))^{*c} = (\langle \{b, a\}, \emptyset, \emptyset \rangle)^{*c} = \langle \{b, c\}, \emptyset, \emptyset \rangle$.

Conclusion

We defined a new types of neutrosophic crisp closed sets and limit points in neutrosophic crisp topological space namly [neutrosophic crisp Gem sets and neutrosophic crisp Turig points] respactvely , we stady their properties in details and we also use it to introduce the some of topological concepts as : neutrosophic crisp closed (open) sets , neutrosophic crisp closure , neutrosophic crisp interior , neutrosophic crisp extrior and neutrosophic crisp boundary which are fundamental for further reserch on neutrosophic crisp topology and will setrengthen the foundations of theory of neutrosophic topological spaces .

We expect, this paper will promote the future study on neutrosophic crisp topological spaces and many other general frameworks .

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