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Neutrosophic Data Envelopment Analysis: An Application to Regional Hospitals in Tunisia

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Abstract: In many real-life situations, decision-making units (DMUs)—such as production processes or manufacturing or service systems—involve data related to inputs and outputs that are volatile, imprecise, or even missing. This makes it difficult to measure these DMUs' efficiency. In this context, a data envelopment analysis (DEA) is a powerful methodology to facilitate this measurement, but this is also sensitive to data: any noise or error in the data measurement can easily cause non-applicable or insignificant results. The neutrosophic theory has demonstrated its superiority over other approaches and theories in handling this type of data, and especially in its capability to consider indeterminate data. However, in the DEA context, the use of this theory remains limited to a few theoretical works. In order to filling this gap, the present paper aims to highlight the neutrosophic DEA in a real-life application. Two different neutrosophic approaches, or namely, the ranking and parametric approaches, are adjusted then applied to measure and evaluate the efficiency of 32 regional hospitals in Tunisia. These results allow a comparison of these two approaches, but more importantly, they reveal the desired efficiency measurement that permits inefficient hospitals' necessary actions. Consequently, indeterminate inputs and outputs are no longer a handicap in using the DEA.

Keywords: data envelopment analysis; indeterminate data; neutrosophic sets; hospital efficiency

1. Introduction

All organizations, whether governmental or private, need an accurate performance assessment for development, growth, and sustainability. In fact, in today's competitive environment, these organizations face pressure to convert inputs into outputs as cheaply as possible (at a given level of quality and quantity). This pressure encourages them to be efficient. Precisely, in the public sector, where the usual disciplines of a competitive market are absent, one of the key roles of government is to provide public goods and services. So that, identifying efficient providers can enhance efficiency by allowing the recognition and spread of good practice.

In seeking to evaluate the technical efficiency of a set of decision-making units (DMUs), Charnes et al. [1] proposed the data envelopment analysis (DEA) methodology. Subsequently, this technique has been used in a variety of models and applications, or in more than 4,000 publications as noted by Emrouznejad et al. [2]. In presence of several inputs and outputs, the DEA essentially uses linear programming to find a best-practice frontier for efficient DMUs that envelops all other inefficient DMUs. This methodology is especially popular because it does not require any specified production function, and can simultaneously consider many inputs and outputs.

The original DEA methodology fundamentally assumes that inputs and outputs are measured with crisp, positive values on a ratio scale, and all the required data are available. As its name indicates, this methodology is highly sensitive to data: any noise or error in data measurement can easily cause non-applicable or insignificant results. Therefore, a key to the DEA's success involves accurately measuring all factors, including inputs and outputs. However, the data related to inputs and outputs in many real-life situations—such as in production processes or manufacturing or service systems—are volatile, imprecise, or even missing. Therefore, it is desirable to use theories and methods that can handle this kind of data.

Among many approaches, such as: (1) stochastic methods; Cooper et al. [3] treated the topic of stochastic characterizations of efficiency and inefficiency in DEA using chance constrained programming formulations and constructs centered on congestion as one form of inefficiency. Khodabakhshi et al. [4] developed an input-oriented super-efficiency measure in stochastic data envelopment analysis. (2) interval DEA models; Entani and Tanaka [5] presented a method in order to improve the efficiency interval of a DMU by adjusting its given inputs and outputs. Smirlis et al. [6] introduced an approach based on interval DEA that allows the evaluation of the units with data. Jahanshahloo et al. [7]], developed an interval DEA model to obtain an efficiency interval consisting of evaluations from both the optimistic and the pessimistic viewpoints. (3) fuzzy theory; introduced by Zadeh [8], it has been mostly applied to handle imprecise, uncertain, or incomplete data in DEAs. Sengupta [9] is the first who explored the use of fuzzy set-theory in the context of data envelopment analysis. Kao and Liu [10] presented a procedure to measure the efficiencies of DMUs with fuzzy observations. authors transformed a fuzzy DEA model to a family of conventional crisp DEA models by applying the α-cut approach. Wang et al. [11] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic to deal with fuzziness in input and output data in DEA. Zerafat et al. [12] introduced the concept of ''local α -level'' to develop a multi-objective linear programming to measure the DEA efficiency of DMUs under uncertainty. Agarwal [13] proposed a fuzzy DEA model based on α-cut approach to deal with the efficiency measuring and ranking problem. Kumar [14] applied fuzzy data envelopment analysis in assessing the productivity of banks. According to Hatami-Marbini et al. [15], DEA approaches using fuzzy theory can be classified into four primary categories, while Emrouznejad et al. [16] presented a taxonomy of the fuzzy DEA methods, with a classification scheme that includes six categories.

Although the fuzzy set theory has been introduced as a powerful tool to quantify vague data, a key inadequacy exists in these past methodologies. A critical problem is that fuzziness is insufficient to consider the degree of information certainty when handling real data. Smarandache [18] recently introduced the neutrosophic theory as a generalization of fuzzy theory. As this can handle vague, imprecise, incomplete, as well as indeterminate data, the neutrosophic theory is considered closer to human thinking due to its better simulation of human decision-making processes by considering indeterminate data. In fact, each element of a neutrosophic set has truth, indeterminacy, and falsity membership functions. Since Smarandache's introduction of the neutrosophic set concept, many different sets have been proposed, with the single value neutrosophic set introduced by Wang et al. [19] as the most popular. Single-valued neutrosophic numbers present a special case involving single-valued neutrosophic sets, and are important in neutrosophic, multi-attribute decision-making problems because they effectively describe an ill-known quantity (Deli and Şubaş, [20]).

The neutrosophic set theory has since been applied in many mathematical programming and multi-criteria decision-making methods, such as the following: linear programming [Abdel-Nasser et al. [21], Abdel-Basset [22]], non-linear programming (Ye et al. [23]), the analytic hierarchy process (Abdel-Basset et al. [24]), goal programming [Pramanik [25], Pramanik and Banerjee [26]], analytic hierarchy process combined with preference ranking organization method for enrichment evaluations type II method (Abdel-Basset et al. [27]), and the technique for order preference by similarity to an ideal solution (Biswas et al. [27]), among others. Abdel-Nasser and Hagar [28] also present some earlier works using multi-criteria decision-making methods in a neutrosophic environment. Further, the concept of neutrosophic sets and its extensions have been applied in a variety of fields, including computer science (Ali and Smarandache, [29]), mathematics (Salama and Alblowi, [30]), and medicine (Abdel-Basset et al. [31]).

In the DEA context, few studies to the best of our knowledge have addressed neutrosophic data. Edalatpanah [32] presented a brief DEA model with neutrosophic inputs and outputs, and suggested that the score function developed by Despotis and Smirlis [33] be used to transform the model into a crisp DEA model and solve it using any conventional method. Abdelfattah [34] also presented a DEA model with all neutrosophic inputs and outputs; the author solved this model by developing a parametric approach based on what he called the "degrees of variation" in a neutrosophic number. However, these two studies are only theoretical, and have not applied their neutrosophic DEA models to real examples to further demonstrate the importance of this research axis. Therefore, this paper aims to highlight the neutrosophic DEA approach in a real-life application through an efficiency evaluation of Tunisian regional hospitals with indeterminate data.

The remainder of the paper is organized as follows: Section 2 introduces the neutrosophic DEA model and the two approaches that will follow for its resolution. Section 3 presents the main body of the paper and its data, data adjustment, results, and analysis related to the application case. Section 4 provides a summary and the research's final conclusions.

2. Methodology

2.1. Neutrosophic DEA model

Charnes et al. [1] developed the first DEA model to measure the relative efficiency of a set of homogenous DMUs under the assumption of constant returns to scale. First, let x_{ij} and y_{rj} denote the inputs and outputs of a DMU j, respectively, with m inputs, s outputs, and n DMUs. The output-oriented DEA model measuring the efficiency of a given DMU k is:

$$\begin{aligned} & Min\ E_k = \sum_{i=1}^{m} v_i x_{ik} \,/ \sum_{r=1}^{s} u_r y_{rk} \\ & Subject\ to: \\ & \sum_{i=1}^{m} v_i x_{ij} \,/ \sum_{r=1}^{s} u_r y_{rj} \geq 1, j = 1, 2, \dots, n; \\ & u_r, v_i \geq 0, \forall\, r, \end{aligned} \tag{1}$$

where u_r indicates the weight assigned to the output r, and v_i is the weight assigned to the input i.

Model (1) is a fractional programming model converted into linear programming, as follows:

$$\begin{aligned} & Min\ E_k = \sum_{i=1}^{m} v_i x_{ik} \\ & Subject\ to: \\ & \sum_{r=1}^{s} u_r y_{rk} = 1 \\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0, j = 1, 2, \dots, n; \\ & u_r, v_i \geq 0, \forall\, r, i \end{aligned} \tag{2}$$

If any of this model's observation data related to inputs and/or outputs is imprecise, uncertain, or indeterminate, then the efficiency of the DMU k will be misleading. Additionally, if this DMU lies on the efficient production function, it will reflect a doubtful reference unit for the other inefficient DMUs. A powerful approach to address this kind of problem involves relying on the neutrosophic set theory.

Assuming inputs and outputs are neutrosophic, they can be represented by triangular neutrosophic numbers, while the variables u_r and v_i are real numbers; thus, Model (2) will be written as follows:

$$Min\ \tilde{E}_k = \sum_{i=1}^{m} v_i \tilde{x}_{ik}$$

$$Subject\ to:$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rk} = 1 \tag{3}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, j = 1, 2, \dots, n;$$

$$u_r, v_i \geq 0, \forall\, r, i$$

where $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are triangular neutrosophic numbers, such that:

$$\tilde{x}_{ij} = \langle (x_{ij1}, x_{ij2}, x_{ij3}), t_{\tilde{x}_{ij}}, d_{\tilde{x}_{ij}}, f_{\tilde{x}_{ij}} \rangle$$

$$\tilde{y}_{rj} = \langle (y_{rj1}, y_{rj2}, y_{rj3}), t_{\tilde{y}_{rj}}, d_{\tilde{y}_{rj}}, f_{\tilde{y}_{rj}} \rangle$$

where x_{ij1}, x_{ij2}, and x_{ij3} denote the lower bound, median value, and upper bound of $\tilde{x}_{ij}$, respectively; $t_{\tilde{x}_{ij}}, d_{\tilde{x}_{ij}}$, and $f_{\tilde{x}_{ij}}$ indicate the degrees of truth, indeterminacy, and falsity for $\tilde{x}_{ij}$, respectively. Subsequently, $\tilde{y}_{rj}$ is defined in a similar manner.

As Model (3) is a neutrosophic DEA model that cannot be solved using typical techniques, the author suggests using the following approaches while introducing some modifications that make them applicable in the proposed model.

2.2. Ranking approach

As a first alternative, we consider the approach from work by Abdel-Basset et al. [22], which was specifically developed to address neutrosophic linear programming models. This method suggests that each trapezoidal neutrosophic number $\tilde{a}$ be converted into its equivalent crisp value using the following ranking function:

$$R = \left(\frac{a^l + a^u + 2(a^{m1} + a^{m2})}{2} \right) + (t_{\tilde{a}} - d_{\tilde{a}} - f_{\tilde{a}}) \tag{4}$$

Note that $\tilde{a} = \langle (a^l, a^{m1}, a^{m2}, a^u), t_{\tilde{a}}, d_{\tilde{a}}, f_{\tilde{a}} \rangle$ is a trapezoidal neutrosophic number, where a^l, a^{m1}, a^{m2}, and a^u are the lower bound, first and second median values, and the upper bound of $\tilde{a}$, respectively; $t_{\tilde{a}}, d_{\tilde{a}}$, and $f_{\tilde{a}}$ are the degrees of truth, indeterminacy, and falsity for the trapezoidal number; and $(t_{\tilde{a}} - d_{\tilde{a}} - f_{\tilde{a}})$ indicates the degree of confirmation.

As the DEA model can be transformed as shown into a linear programming model, we can apply this ranking function to solve Model (3). Accordingly, the input and output values in this work should be triangular neutrosophic numbers, and thus, we propose the following ranking function:

$$= \left(\frac{a^l + 2a + a^u}{4} \right) + (t_{\tilde{a}} - d_{\tilde{a}} - f_{\tilde{a}}) \tag{5}$$

Applying this ranking function to Model (3) obtains the following crisp explicit model; standard methods are then used to calculate the optimal solution:

$$Min\ \tilde{E}_k = \sum_{i=1}^{m} v_i \left[{}^1\!/_4 \left(x_{ik1} + 2x_{ik2} + x_{ik3} \right) + \left(t_{\tilde{x}_{ik}} - d_{\tilde{x}_{ik}} - f_{\tilde{x}_{ik}} \right) \right] \tag{6}$$

$Subject\ to$:

$$\sum_{r=1}^{s} u_r\left[{}^1/_4\left(y_{rk1}+2y_{rk2}+y_{rk3}\right)+\left(t_{\tilde{y}_{rk}}-d_{\tilde{y}_{rk}}-f_{\tilde{y}_{rk}}\right)\right]=1$$

$$\sum_{r=1}^{s} u_r\left[{}^1/_4\left(y_{rj1}+2y_{rj2}+y_{rj3}\right)+\left(t_{\tilde{y}_{rj}}-d_{\tilde{y}_{rj}}-f_{\tilde{y}_{rj}}\right)\right] - \sum_{i=1}^{m} v_i\left[{}^1/_4\left(x_{ij1}+2x_{ij2}+x_{ij3}\right)+\left(t_{\tilde{x}_{ij}}-d_{\tilde{x}_{ij}}-f_{\tilde{x}_{ij}}\right)\right]\le 0, j=1,2,\dots,n;$$

$u_r, v_i \ge 0, \forall\, r, i$

2.3. Parametric approach

All input and output data in Model (3) should be triangular neutrosophic numbers. Unlike the ranking approach, Abdelfattah's [34] proposed parametric approach consists of transforming these data into intervals rather than crisp values by considering the decision-makers' levels of acceptance, indeterminacy, and rejection toward the data. This approach essentially determines the degrees of variation for every single neutrosophic input or output, given by the following equation:

$$\theta_{\tilde{a}}=\frac{1}{4}\left[\frac{\alpha}{t_{\tilde{a}}}+2\frac{(1-\beta)}{1-d_{\tilde{a}}}+\frac{(1-\gamma)}{1-f_{\tilde{a}}}\right];\quad \theta_{\tilde{a}}\in[0,1]; \tag{7}$$

where $t_{\tilde{a}}, d_{\tilde{a}}$, and $f_{\tilde{a}}$ indicate the degrees of truth, indeterminacy, and falsity for the triangular neutrosophic number $\tilde{a}$, respectively; α denotes the minimal degree of acceptance, or $\alpha\in[0,t_{\tilde{a}}]$; β denotes the maximal degree of indeterminacy, or $\beta\in[d_{\tilde{a}},1]$; and γ denotes the maximal degree of rejection, or $\gamma\in[f_{\tilde{a}},1]$.

Input and output values are then converted into their equivalent intervals with the following equation:

$$\tilde{a}=[a^l,a^u]=[a_1+(a_2-a_1)\theta_{\tilde{a}}, a_3-(a_3-a_2)\theta_{\tilde{a}}] \tag{8}$$

where a_1, a_2, and a_3 are the lower bound, median value, and upper bound of $\tilde{a}$, respectively.

According to this approach, Model (3) can then be transformed into two sub-models. Note that Abdelfattah [34] adopted an input-oriented DEA model, while this paper adopts an output-oriented DEA model, as its application will require. Thus, Model (3) is transformed into the following two sub-models (9a) and (9b), representing the most favorable (maximal) efficiency and the least favorable (minimal) efficiency, respectively:

$$Min\ (E_k)_\theta^u=\sum_{i=1}^{m} v_i[x_{ik1}+(x_{ik2}-x_{ik1})\theta_{\tilde{x}_{ik}}]$$

$Subject\ to$:

$$\begin{aligned}&\sum_{r=1}^{s} u_r[y_{rk3}-(y_{rk3}-y_{rk2})\theta_{\tilde{y}_{rk}}]=1;\\ &\sum_{i=1}^{m} v_i\left[x_{ik1}+(x_{ik2}-x_{ik1})\theta_{\tilde{x}_{ik}}\right]-\sum_{r=1}^{s} u_r[y_{rk3}-(y_{rk3}-y_{rk2})\theta_{\tilde{y}_{rk}}]\ge 0;\\ &\sum_{i=1}^{m} v_i\left[x_{ij3}-(x_{ij3}-x_{ij2})\theta_{\tilde{x}_{ij}}\right]-\sum_{r=1}^{s} u_r[y_{rj1}+(y_{rj2}-y_{rj1})\theta_{\tilde{y}_{rj}}]\ge 0, j=1,2,\dots,n,\quad j\ne k;\\ &u_r, v_i\ge 0, \forall\, r, i\end{aligned} \tag{9 a}$$

$$Min\ (E_k)^l_\theta = \sum_{i=1}^{m} v_i[x_{ik3} - (x_{ik3} - x_{ik2})\theta_{\tilde{x}_{ik}}]$$

$$Subject\ to:$$

$$\sum_{r=1}^{s} u_r[y_{rk1} + (y_{rk2} - y_{rk1})\theta_{\tilde{y}_{rk}}] = 1$$

$$\sum_{i=1}^{m} v_i[x_{ik3} - (x_{ik3} - x_{ik2})\theta_{\tilde{x}_{ik}}] - \sum_{r=1}^{s} u_r[y_{rk1} + (y_{rk2} - y_{rk1})\theta_{\tilde{y}_{rk}}] \geq 0 \quad (9\text{ b})$$

$$\sum_{i=1}^{m} v_i\left[x_{ij1} + (x_{ij2} - x_{ij1})\theta_{\tilde{x}_{ij}}\right] - \sum_{r=1}^{s} u_r[y_{rj3} - (y_{rj3} - y_{rj2})\theta_{\tilde{y}_{rj}}] \geq 0, j = 1, 2, \dots, n, \qquad j \neq k$$

$$u_r, v_i \geq 0, \forall\ r, i$$

$$\theta_{\tilde{x}_{ij}} = \frac{1}{4}\left[\frac{\alpha}{t_{\tilde{x}_{ij}}} + 2\frac{(1-\beta)}{1-d_{\tilde{x}_{ij}}} + \frac{(1-\gamma)}{1-f_{\tilde{x}_{ij}}}\right];\ \theta_{\tilde{y}_{rj}} = \frac{1}{4}\left[\frac{\alpha}{t_{\tilde{y}_{rj}}} + 2\frac{(1-\beta)}{1-d_{\tilde{y}_{rj}}} + \frac{(1-\gamma)}{1-f_{\tilde{y}_{rj}}}\right]$$

$$\alpha \in \left[0, min\left\{t_{\tilde{x}_{ij}}, t_{\tilde{y}_{rj}}\right\}\right];\ \beta \in \left[max\left\{d_{\tilde{x}_{ij}}, d_{\tilde{y}_{rj}}\right\}, 1\right];\ \gamma \in \left[max\left\{f_{\tilde{x}_{ij}}, f_{\tilde{y}_{rj}}\right\}, 1\right]$$

After a decision-maker sets specific values of α, β, and γ—representing his or her minimal degree of acceptance, maximal degree of indeterminacy, and maximal degree of rejection, respectively—Models (9a) and (9b) will yield bounded intervals of efficiency scores $\left[(E_k)^l_{\theta_i}, (E_k)^u_{\theta_i}\right]$ for all evaluated DMUs.

3. An Application to Evaluate the Efficiency of Regional Hospitals in Tunisia

Providing suitable healthcare services is key for every society's well-being. Tunisia considers the health sector as a national priority, and invested 7% of its 2014 gross domestic product in its healthcare industry.1 This percentage is higher than the minimum 5% threshold recommended by the World Health Organization, and is equivalent to that of upper-middle-income countries. Tunisia's public health facilities are classified according to their mission, equipment, technical level, and territorial competence, categorized as: basic health centers, district hospitals, regional hospitals, and university hospital centers.

As this paper is only concerned with regional hospitals, we attempt to measure their ability to efficiently use minimum resources (inputs) to produce suitable healthcare services (outputs) using the DEA. Literature has similarly applied the DEA in this type of efficiency measurement; specifically, Kohl et al. [35] provide a noteworthy review of this issue. As some observations are not available and others are not "precise," this paper applies the concept of indeterminacy, and therefore, the approaches described in the previous section.

3.1. Data

This study evaluates all 32 regional hospitals in Tunisia, with data collected from the Ministry of Public Health's 2015 health map.2

The selection of inputs and outputs to be considered is typically a subject of debate. For example, Ozcan [36] suggested that inputs include beds, a weighted service-mix, full-time equivalents, and operations expenses, and that outputs include case-mix-adjusted admissions and outpatient visits. Azreena et al. [37] systematically reviewed hospitals' inputs and outputs in measuring efficiency

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[1] https://www.who.int/countries/tun/en visited on 15-July-2019

[2] http://www.santetunisie.rns.tn/images/docs/anis/stat/cartesanitaire2015.pdf visited and downloaded on 18-July-2019

using a DEA. Regarding this issue, Dyson et al. [38] stated that using significant numbers of inputs and outputs does not necessarily garner better results. These authors posit that the most important factor is the number of DMUs, as there should always be more than $2 \times (number\ of\ inputs + number\ of\ outputs)$. This study respects this rule, as three outputs and only one input are considered for the 32 regional hospitals, as follows:

Type	Name	Explanation
Input	Operating Budget (OB)	The hospital's annual expenses, coming from: - The state's budget in terms of salaries - Contributions from the public health insurance fund (CNAM) - Net revenues
Output 1	Admissions	The admissions of hospitalized patients at a hospital for a given period. As hospital statistics do not distinguish between the number of admissions and the number of entries, the same patient can be re-hospitalized for the considered period and generate several entries.
Output 2	Outpatient Visits	The number of times that a patient is not hospitalized overnight, but visits the hospital for diagnosis or treatment.
Output 3	Emergency Visits	The number of cases calling for immediate action as registered by the hospital's emergency room/department.

Tables 1 and 2 present the data regarding the considered inputs and outputs, respectively.

Table 1. Input data: 2015 operating budget of Tunisian regional hospitals (in TND)

DMU	DMU Name: Hospital	Salaries	CNAM	Net Revenue	Total OB
1	Mahmoud El Matri de l'Ariana	200,000	1,796,390	707,249	2,703,639
2	Khair-Eddine	500,000	1,333,887	233,547	2,067,434
3	Hôpital Ben Arous	100,000	3,611,455	1,529,633	5,241,088
4	Menzel Bourguiba	*	7,236,055	1,567,819	8,803,874
5	Nabeul	200,000	2,331,000	1,138,213	3,669,213
6	Menzel Témime	**0**	3,907,115	1,191,333	5,098,448
7	Zaghouan	200,000	2,656,103	864,525	3,720,628
8	Jendouba	500,000	4,393,450	1,334,386	6,227,836
9	Tabarka	*	*	1,400,000	1,400,000
10	Béja	400,000	5,873,920	1,078,012	7,351,932
11	Medjez El Bab	400,000	1,568,142	426,514	2,394,656
12	M'hamed Bourguiba du Kef	401,000	5,654,441	1,008,723	7,064,164
13	Siliana	**0**	4,869,819	925,074	5,794,893
14	Kasserine	1,000,000	5,270,338	1,890,662	8,161,000
15	M'Saken	200,000	1,586,683	1,020,266	2,806,949
16	Moknine	200,000	2,717,575	688,340	3,605,915
17	Haj Ali Soua de Ksar Hellal	**0**	2,025,418	1,053,743	3,079,161
18	Kerkennah	**0**	2,551,472	269,535	2,821,007

19	Jebeniana	300,000	1,576,958	608,216	2,485,174
20	Mahres	200,000	1,377,638	529,700	2,107,338
21	Houcine Bouzaiene de Gafsa	400,000	3,200,000	922,708	4,522,708
22	Metlaoui	100,000	2,216,796	337,818	2,654,614
23	Tozeur	100,000	4,032,146	564,305	4,696,451
24	Sidi Bouzid	**0**	6,208,290	1,387,163	7,595,453
25	Mohamed Ben Sassi de Gabès	700,000	8,249,323	2,380,640	11,329,963
26	Kébili	300,000	4,365,460	901,949	5,567,409
27	Habib Bourguiba de Médenine	**0**	2,884,710	1,754,319	4,639,029
28	Sadok Mokadem de Jerba	**0**	4,967,925	1,569,452	6,537,377
29	Zarzis	**0**	2,858,210	1,028,585	3,886,795
30	Ben Guerdenne	300,000	2,016,560	822,980	3,139,540
31	Tataouine	500,000	2,473,696	870,845	3,844,541
32	Nefta	**0**	548,000	560,978	1,108,978
Minimum (missing values and zeros are not included)		100,000	548,000		
Maximum (missing values are not included)		1,000,000	8,249,323		
Median (missing values and zeros are not included)		300,000	2,858,210		
Median-minimum		200,000	2,310,210		
Maximum-medium		700,000	5,391,113		

Table 2. Output data

DMU	DMU Name: Hospital	Admissions	Outpatient Visits	Emergency Visits
1	Mahmoud El Matri de l'Ariana	4,544	58,233	26,500
2	Khair-Eddine	607	59,349	31,623
3	Hôpital Ben Arous	10,162	96,391	72,402
4	Menzel Bourguiba	11,720	64,402	71,357
5	Nabeul	10,845	22,203	62,534
6	Menzel Témime	10,105	36,757	73,075
7	Zaghouan	6,933	44,853	54,254
8	Jendouba	15,238	80,248	98,661
9	Tabarka	2,229	10,500	40,073
10	Béja	11,115	54,742	66,014
11	Medjez El Bab	2,479	29,175	38,695
12	M'hamed Bourguiba du Kef	11,812	64,752	84,222
13	Siliana	13,460	73,979	56,981
14	Kasserine	27,006	61,565	115,607
15	M'Saken	2,459	60,712	76,092
16	Moknine	4,432	38,672	52,532
17	Haj Ali Soua de Ksar Hellal	3,353	34,494	73,422
18	Kerkennah	2,205	1,557	16,190

19	Jebeniana	3,483	33,232	42,993
20	Mahres	2,722	34,609	26,950
21	Houcine Bouzaiene de Gafsa	15,400	72,357	143,696
22	Metlaoui	3,121	31,753	29,854
23	Tozeur	7,598	25,308	45,724
24	Sidi Bouzid	15,040	76,156	70,796
25	Mohamed Ben Sassi de Gabès	26,509	101,642	123,141
26	Kébili	11,608	33,938	48,268
27	Habib Bourguiba de Médenine	14,005	60,539	67,476
28	Sadok Mokadem de Jerba	18,010	33,846	58,814
29	Zarzis	11,070	26,095	39,655
30	Ben Guerdenne	6,344	30,529	44,110
31	Tataouine	8,498	24,537	42,080
32	Nefta	935	13,256	21,890

3.2. De-neutrosophizing the input data

Table 1 reveals that salary values are missing as related to the Menzel Bourguiba (DMU 4) and Tabarka (DMU 9) hospitals. Further, the latter exhibits another missing value related to the annual amount received from the CNAM public insurance fund. Additionally, the same table indicates that various hospitals—represented by DMUs 6, 13, 17, 18, 24, 27, 28, 29, and 32—recorded zero amounts for annual salaries. This data cannot be correct, as a government can delay remunerations in certain difficult circumstances, but cannot refuse to give salaries for an entire year. Hence, the OB information is incomplete, imprecise, and subsequently indeterminate, and contrary to the outputs noted in Table 2 as crisp values, the input OB for each of the previously mentioned DMUs will be treated as neutrosophic data.

By choosing to represent the neutrosophic data as triangular neutrosophic numbers, the lower bounds, median values, and upper bounds should be set. As they are not available, we calculate them as follows:

The lower bounds are the same as the obtained total values in Table 1:

$$OB^l = Salaries + CNAM + Net\ revenue$$

The median value is:

$$OB^m = OB^l + (median - minimum)$$

The upper bound is:

$$OB^u = OB^l + (maximum - median)$$

Table 3 presents all obtained values of these bounds for the considered DMUs. Further, the same table presents the degrees of truth, indeterminacy, and falsity—or $t_{\widetilde{OB}}$, $d_{\widetilde{OB}}$, and $f_{\widetilde{OB}}$, respectively—that decision-maker(s) should give subjectively.

Table 3. Bounds; degrees of truth, indeterminacy, and falsity; and input data degrees of variation

DMU	$\widetilde{OB} =< (OB^l, OB^m, OB^u), t_{\widetilde{OB}}, d_{\widetilde{OB}}, f_{\widetilde{OB}} >$						Degrees of Variation		
	OB^l	OB^m	OB^u	$t_{\widetilde{OB}}$	$d_{\widetilde{OB}}$	$f_{\widetilde{OB}}$	(0; 1; 1)	(0,7; 0,3; 0,4)	(0,4; 0,6; 0,7)
4	8,803,874	9,003,874	9,503,874	0.9	0.1	0.3	0	0.798	0.440
6	5,098,448	5,298,448	5,798,448	0.8	0.2	0.2	0	0.844	0.469
9	1,400,000	3,910,210	7,491,113	0.7	0.3	0.4	0	1	0.554

13	5,794,893	5,994,893	6,494,893	0.8	0.2	0.2	0	0.844	0.469
17	3,079,161	3,279,161	3,779,161	0.8	0.2	0.2	0	0.844	0.469
18	2,821,007	3,021,007	3,521,007	0.8	0.2	0.2	0	0.844	0.469
24	7,595,453	7,795,453	8,295,453	0.8	0.2	0.2	0	0.844	0.469
27	4,639,029	4,839,029	5,339,029	0.8	0.2	0.2	0	0.844	0.469
28	6,537,377	6,737,377	7,237,377	0.8	0.2	0.2	0	0.844	0.469
29	3,886,795	4,086,795	4,586,795	0.8	0.2	0.2	0	0.844	0.469
32	1,108,978	1,308,978	1,808,978	0.8	0.2	0.2	0	0.844	0.469

The ranking approach can be now applied. However, the degrees of variation for the obtained neutrosophic numbers should be calculated for the parametric approach, and it is only sufficient to set a single value each for $\alpha \in [0, 0.7]$, $\beta \in [0.3, 1]$, and $\gamma \in [0.4, 1]$. We respect these ranges by choosing to consider three different values in the triplet (α, β, γ). This will yield superior in-depth analyses and interpretations of the obtained efficiencies.

Table 3 displays the obtained degrees of variation, and easily reveals that all degrees of variation for $(\alpha, \beta, \gamma) = (0; 1; 1)$ equal zero. This parallels the definition of degrees of variation given by Abdelfattah [34], in which this degree is null when decision-maker chooses to set the degree of acceptance at its minimum ($\alpha = 0$) and the degrees of indeterminacy and rejection at their maximum ($\beta = 1$ and $\gamma = 1$). The opposite case is also verified; in fact, DMU 9 has a recorded degree of variation that equals 1 when the decision-maker sets the acceptance degree at its maximum ($\alpha = 0.7$) and the degrees of indeterminacy and rejection at their minimum ($\beta = 0.3$ and $\gamma = 0.4$). Only this DMU has a degree of variation that equals 1 because this is the only one with the same time $t_{\widetilde{OB}} = 0.7$, $d_{\widetilde{OB}} = 0.3$, and $f_{\widetilde{OB}} = 0.4$.

Once the degrees of variation are set, the parametric approach can be applied to convert triangular neutrosophic values related to the input OB into their corresponding interval ranges. The ranking approach does not need these degrees of variation, as it relies only on the availability of the bounds and degrees of truth, indeterminacy, and falsity. Table 4 illustrates the intervals and crisp values of inputs yielded through the parametric and ranking approaches, respectively.

Table 4. De-neutrosophized input data

DMU	Parametric Approach			Ranking Approach
	(0; 1; 1)	(0,7; 0,3; 0,4)	(0,4; 0,6; 0,7)	
4	[8,803,874; 9,503,874]	[8,963,398; 9,105,064]	[8,891,969; 9,283,636]	9,078,875
6	[5,098,448; 5,798,448]	[5,267,198; 5,376,573]	[5,192,198; 5,564,073]	5,373,448
9	[1,400,000; 7,491,113]	3,910,210	[2,789,581; 5,508,827]	4,177,883
13	[5,794,893; 6,494,893]	[5,963,643; 6,073,018]	[5,888,643; 6,260,518]	6,069,893
17	[3,079,161; 3,779,161]	[3,247,911; 3,357,286]	[3,172,911; 3,544,786]	3,354,161
18	[2,821,007; 3,521,007]	[2,989,757; 3,099,132]	[2,914,757; 3,286,632]	3,096,007
24	[7,595,453; 8,295,453]	[7,764,203; 7,873,578]	[7,689,203; 8,061,078]	7,870,453
27	[4,639,029; 5,339,029]	[4,807,779; 4,917,154]	[4,732,779; 5,104,654]	4,914,029
28	[6,537,377; 7,237,377]	[6,706,127; 6,815,502]	[6,631,127; 7,003,002]	6,812,377
29	[3,886,795; 4,586,795]	[4,055,545; 4,164,920]	[3,980,545; 4,352,420]	4,161,795
32	[1,108,978; 1,808,978]	[1,277,728; 1,387,103]	[1,202,728; 1,574,603]	1,383,978

Table 4 demonstrates that the largest-interval input values are obtained when $(\alpha, \beta, \gamma) = (0, 1, 1)$, and the smallest intervals are obtained when $(\alpha, \beta, \gamma) = (0{,}7; 0{,}3; 0{,}4)$. Moreover, all input values obtained using the ranking approach are included in their corresponding intervals obtained by the parametric approach, except for DMU 9, when $(\alpha, \beta, \gamma) = (0{,}7; 0{,}3; 0{,}4)$. This is a favorable sign, indicating that the efficiency intervals and scores yielded using the two approaches will likely be very close.

3.3. Results

Table 5 provides the results from applying Models (6), (9a), and (9b) to obtain efficiency scores for Tunisia's 32 regional hospitals.

Table 5. Efficiency scores of Tunisian regional hospitals using the two approaches

DMU	**Parametric Approach**					**Ranking Approach**	
	Scores			Ranking Index	Rank	Scores	Rank
	(0; 1; 1)	(0,7; 0,3; 0,4)	(0,4; 0,6; 0,7)				
1	0.950	0.950	0.950	0.952	5	0.950	5
2	1.000	1.000	1.000	1.000	1	1.000	1
3	0.880	0.880	0.880	0.884	6	0.880	6
4	[0.395, 0.426]	[0.412, 0.419]	[0.404, 0.422]	0.336	31	0.414	30
5	0.868	0.868	0.868	0.872	7	0.868	7
6	[0.512, 0.582]	[0.552, 0.563]	[0.533, 0.572]	0.516	24	0.552	24
7	0.658	0.658	0.658	0.643	16	0.658	16
8	0.765	0.765	0.765	0.764	11	0.765	11
9	[0.168, 0.901]	0.323	[0.229, 0.452]	0.348	30	0.302	31
10	0.455	0.455	0.455	0.389	29	0.455	29
11	0.594	0.594	0.594	0.567	21	0.594	21
12	0.535	0.535	0.535	0.493	27	0.535	26
13	[0.664, 0.744]	[0.710, 0.723]	[0.688, 0.732]	0.695	13	0.710	13
14	0.972	0.972	0.972	0.974	4	0.972	4
15	1.000	1.000	1.000	1.000	1	1.000	1
16	0.538	0.538	0.538	0.497	26	0.538	25
17	[0.611, 0.750]	[0.688, 0.712]	[0.652, 0.728]	0.669	15	0.689	14
18	[0.184, 0.230]	[0.209, 0.217]	[0.197, 0.222]	0.040	32	0.209	32
19	0.654	0.654	0.654	0.639	17	0.654	17
20	0.726	0.726	0.726	0.720	12	0.726	12
21	1.000	1.000	1.000	1.000	1	1.000	1
22	0.561	0.561	0.561	0.525	23	0.561	23
23	0.475	0.475	0.475	0.415	28	0.475	28
24	[0.555, 0.606]	[0.584, 0.593]	[0.571, 0.598]	0.554	22	0.585	22
25	0.687	0.687	0.687	0.677	14	0.687	15
26	0.612	0.612	0.612	0.589	19	0.612	19
27	[0.770, 0.887]	[0.836, 0.855]	[0.806, 0.869]	0.820	8	0.837	8
28	[0.731, 0.809]	[0.776, 0.789]	[0.755, 0.798]	0.765	9	0.776	10
29	[0.709, 0.836]	[0.781, 0.802]	[0.747, 0.817]	0.764	10	0.781	9
30	0.601	0.601	0.601	0.575	20	0.601	20
31	0.649	0.649	0.649	0.633	18	0.649	18

32	[0.404, 0.658]	[0.526, 0.571]	[0.464, 0.607]	0.503	25	0.528	27

Table 5 reveals that the two applied approaches act only on DMUs with neutrosophic data, as all other DMUs have the same unchanged efficiency scores, with no loss of information for these latter DMUs. Further, these DMUs have exactly the same crisp efficiency score whether yielded using the ranking approach or using the parametric approach for the three considered values of (α, β, γ).

Another inference from Table 5 is that the efficiency scores for all DMUs with neutrosophic input values obtained using the ranking approach include elements of their corresponding interval efficiency scores obtained using the parametric approach. Additionally, the largest efficiency intervals bound by the highest, best efficiencies and lowest, worst efficiencies are obtained when the acceptance degree α is at its minimum and the degrees of indeterminacy and falsity β and γ are at their maximum $(0, 1, 1)$. In contrast, the smallest efficiency intervals bound by the lowest, best and highest, worst efficiencies are obtained when the acceptance degree α is at its maximum and the degrees of indeterminacy and falsity β and γ are at their minimum $(0.7, 0.3, 0.4)$. Another noteworthy observation is that the efficiency score yielded by the ranking approach is equal or nearly equal to the lower value of the interval efficiency scores yielded by the parametric approach when $(\alpha, \beta, \gamma) = (0{,}7; 0{,}3; 0{,}4)$.

On the one hand, the hospitals' best efficiency scores—equal to 1—were achieved by hospitals represented by DMUs 2, 15, and 21. Although these hospitals exhibited relatively small OBs (Table 1), they successfully recorded important numbers, and especially in outpatient and emergency visits (Table 2). On the other hand, the worst efficiency scores were associated with the Kerkennah hospital, represented by DMU 18, with lowest and highest efficiency scores of 0.184 and 0.230 using the parametric approach, respectively. This hospital also had an efficiency score of 0.209 using the ranking approach, and ranked last according to both approaches. Although it has a relatively small OB (Table 4), in terms of this hospital's outputs, it also has relatively few admissions, outpatient visits, and emergency visits (Table 2). Logically, the region is characterized as a small island, which may be among the causes of these results. This research considers Chen and Klein's [39] ranking index in ranking DMUs using the parametric approach.

Although the Mohamed Ben Sassi de Gabès hospital (DMU 25) has generated important records in terms of output, we found it ranked only in the middle, or specifically, 14th and 15th according to the parametric and ranking approaches, respectively, with the same efficiency score of 0.687. This can be explained by the hospital's important OB values, and this can also be partially applied to the Menzel Bourguiba (DMU 4) and Kasserine hospitals (DMU 14).

3.4. Efficiency improvement

Measuring efficiency is a mean rather than a goal, as the ultimate objective involves finding a way to improve efficiency among inefficient DMUs. Among the DEA's strengths is that it conveys how much an inefficient DMU should reduce the quantity of its inputs and/or increase the quantity of its outputs to be relatively more efficient than other DMUs. One way of achieving this involves using a dual model. This study determines the possible improvements that inefficient hospitals can make by using the dual of model (6) obtained by using the ranking approach. This is chosen because only one dual model should be solved rather than two when using the parametric approach; further, the two approaches have yielded nearly the same efficiency scores and DMU rankings. The explicit dual model is as follows:

$$Max\ \phi \qquad (10)$$

$$Subject\ to:$$

$$\phi[{}^{1}/_{4}\,(y_{rk1}+2y_{rk2}+y_{rk3})+(t_{\tilde{y}_{rk}}-d_{\tilde{y}_{rk}}-f_{\tilde{y}_{rk}})]$$

$$-\sum_{j=1}^{n}\lambda_j\left[{}^{1}/_{4}\left(y_{rj1}+2y_{rj2}+y_{rj3}\right)+\left(t_{\tilde{y}_{rj}}-d_{\tilde{y}_{rj}}-f_{\tilde{y}_{rj}}\right)\right]\leq 0, r=1,2,\dots,s;$$

$$\sum_{j=1}^{n}\lambda_j\left[{}^{1}/_{4}\left(x_{ij1}+2x_{ij2}+x_{ij3}\right)+\left(t_{\tilde{x}_{ij}}-d_{\tilde{x}_{ij}}-f_{\tilde{x}_{ij}}\right)\right]-{}^{1}/_{4}\left(x_{ik1}+2x_{ik2}+x_{ik3}\right)$$

$$-\left(t_{\tilde{x}_{ik}}-d_{\tilde{x}_{ik}}-f_{\tilde{x}_{ik}}\right)\leq 0, i=1,2,\dots,m;$$

$$\lambda_j\geq 0, j=1,2,\dots,n;$$

In this model, ϕ is scalar, such that ϕ^{-1} represents the proportional increase that will be simultaneously applied to all outputs of the k^{th} DMU to make it efficient. Thus, the value of ϕ^{-1} obtained from resolving this model defines the efficiency score of the k^{th} DMU. If ($\phi=1$), this DMU is considered efficient, and inefficient otherwise ($\phi>1$); $\phi^{-1}\in[0,1]$.

The previous Section 3.3 measured the 32 regional hospitals' efficiency scores. Only three hospitals—represented by DMUs 2, 15, and 21—were found to be efficient, such that while maintaining their current input and output values, these hospitals can be considered as references for the other inefficient hospital facilities. Table 6 lists the target values of outputs for the 29 inefficient hospitals; in other words, this table provides the possible output adjustments that these latter facilities can apply to achieve perfect efficiency.

Table 6. Target values of outputs for inefficient DMUs to achieve perfect efficiency

DMU	DMU Name: Hospital	DMU of Reference	Benchmark	Target Value		
				Admissions	Outpatient Visits	Emergency Visits
1	Mahmoud El Matri de l'Ariana	2; 21	(0.69; 0.28)	4,784	61,314	62,486
3	Hôpital Ben Arous	2; 21	(0.98; 0.71)	11,551	109,563	133,183
4	Menzel Bourguiba	2; 21	(0.40; 1.82)	28,343	155,748	274,843
5	Nabeul	21	(0.81)	12,494	58,702	116,579
6	Menzel Témime	21	(1.19)	18,297	85,968	170,726
7	Zaghouan	2; 21	(0.33; 0.67)	10,543	68,208	106,955
8	Jendouba	2; 21	(0.20; 1.29)	19,918	104,896	191,053
9	Tabarka	21	(0.92)	14,226	66,840	132,740
10	Béja	2; 21	(0.10; 1.58)	24,404	120,192	230,253
11	Medjez El Bab	2; 15; 21	(0.16; 0.41; 0.20)	4,173	49,113	65,139
12	M'hamed Bourguiba du Kef	2; 21	(0.31; 1.42)	22,084	121,062	214,013
13	Siliana	2; 21	(0.27; 1.22)	18,947	104,138	183,740
14	Kasserine	21	(1.80)	27,789	130,565	259,292
16	Moknine	2; 15; 21	(0.43; 0.18; 0.49)	8,232	71,830	97,573
17	Haj Ali Soua de Ksar Hellal	21	(0.74)	11,421	53,662	106,569
18	Kerkennah	21	(0.68)	10,542	49,532	98,367
19	Jebeniana	2; 15; 21	(0.34; 0.14; 0.31)	5,322	50,776	65,690
20	Mahres	2; 21	(0.53; 0.22)	3,752	47,699	48,823
22	Metlaoui	2; 21	(0.54; 0.34)	5,568	56,647	65,961

23	Tozeur	21	(1.04)	15,992	75,137	149,216
24	Sidi Bouzid	2; 21	(0.17; 1.66)	25,730	130,284	244,398
25	Mohamed Ben Sassi de Gabès	21	(2.51)	38,579	181,264	359,977
26	Kébili	21	(1.23)	18,957	89,071	176,888
27	Habib Bourguiba de Médenine	21	(1.09)	16,732	78,618	156,129
28	Sadok Mokadem de Jerba	21	(1.51)	23,196	108,989	216,444
29	Zarzis	21	(0.92)	14,171	66,583	132,229
30	Ben Guerdenne	2; 21	(0.02; 0.68)	10,554	50,787	99,026
31	Tataouine	21	(0.85)	13,091	61,507	122,149
32	Nefta	15; 21	(0.19; 0.19)	3,370	25,130	41,497

For example, let us consider the Jendouba hospital, represented by DMU 8. Its efficiency score obtained by using the ranking approach is 0.765 (Table 5). This hospital can become efficient by achieving the following: 19,918 admissions, rather than 15,238; 104,896 outpatient visits, rather than 80,248; and 191,053 emergency visits, rather than 98,661 (for current and target values, refer to Tables 2 and 6, respectively). At this point, it should be noted that it is not logical to force people to visit a given hospital to make it efficient. However, an inefficient hospital can be asked to do its best to accommodate more patients based on its actual capacity to do so, given its amount of resources (inputs).

Table 6 also provides the reference hospitals that each inefficient hospital is compared with in calculating their efficiency scores, in addition to their respective possible benchmarks. Let us again consider the Jendouba hospital (DMU 8): its reference hospitals are the Khair-Eddine (DMU 2) and Houcine Bouzaiene de Gafsa hospitals (DMU 21), with a respective benchmark of (0.69; 0.28). Thus, we have:

$$\sum_{r=1}^{3} y_{r8}^{*} = 0.69 \times \sum_{r=1}^{3} y_{r2} + 0.28 \times \sum_{r=1}^{3} y_{r21} \tag{11}$$

where $\sum_{r=1}^{3} y_{r8}^{*}$ denotes the total target output of DMU 8, $\sum_{r=1}^{3} y_{r2}$ is the total current output of DMU 2, and $\sum_{r=1}^{3} y_{r21}$ is the total current output of DMU 21.

4. Conclusions

One requirement in using the DEA methodology to measure efficiency is that all input and output data from each DMU should be available in advance with their crisp values; otherwise, classic DEA models are inapplicable. Many approaches have been developed to handle these types of problems, such as stochastic methods, interval DEA models, and fuzzy theory. However, these approaches do not consider the information's degree of sureness, and the neutrosophic theory demonstrates its power at this moment. In fact, in addition to addressing vague, imprecise, and incomplete data, this theory can also treat indeterminate data.

In the DEA context, only two theoretical works by Edalatpanah [32] and Abdelfattah [34] have handled neutrosophic inputs and outputs. In this paper, however, a real application that consists in measuring and evaluating the efficiency of 32 regional hospitals in Tunisia in a neutrosophic environment. It was demonstrated that neutrosophic DEA can also handle real-world applications.

Two approaches were used: First, the ranking approach as inspired by Abdel-Basset et al. [22] was suggested specifically to solve linear programming models with trapezoidal neutrosophic coefficients. Consequently, a DEA model can be transformed into a linear program; this paper used this approach as a primary alternative given that a trapezoidal neutrosophic number can be reduced to a triangular neutrosophic number. Second, Abdelfattah's [34] parametric approach was used, although this paper used an output-oriented DEA model rather than one that is input-oriented. The two approaches are then compared.

- In terms of their use reveals that the ranking approach is certainly easier, as it does not need the calculation of variation degrees and relies only on the availability of bounds and truth, indeterminacy, and falsity degrees.
- In term of results, the parametric approach is favored, as this approach better interprets the obtained results by providing efficiency scores in ranges bound by the best and worst efficiency scores that a DMU cannot exceed.
- The two approaches act only on DMUs with neutrosophic data; both offer close efficiency scores for these DMUs, or specifically, efficiency scores obtained through the ranking approach are included in the corresponding efficiency intervals obtained through the parametric approach.
- The two approaches give exactly the same efficiency scores for DMUs with crisp data.

From a theoretical perspective, the two approaches applied in this paper measure DMUs' efficiency regardless of the proportion of neutrosophic data. However, such data should be minimized to allow managers to more confidently make their decisions. Moreover, as degrees of truth, indeterminacy, and falsity are subjectively provided, they should be carefully selected.

One noteworthy topic for further research could involve improving one of the existing neutrosophic approaches to solve other DEA models, such as the network DEA. Another adaptation could incorporate another statistical method to better estimate missing and doubtful data bounds' values. Further, a post-analysis of the estimate data could be performed based on obtained efficiency scores and the applied DEA model's adopted orientation.

The DEA method has already demonstrated its power in practice. With the generalization of fuzziness to include neutrosophic logic, this methodology gains the additional capability to evaluate DMUs' performance in terms of their efficiency in real-life applications.

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